Abraham de Moivre

French-born British mathematician **Abraham de Moivre** (May 26, 1667 – November 27, 1754) is best known for the fundamental formula of complex numbers

$$(\cos x + i \sin x)^n = \cos nx + i \sin nx, \text{ where } i = \sqrt{-1},$$

called *de Moivre's theorem.* It is the keystone of analytic trigonometry, linking complex numbers with trigonometry. The result can be used to find explicit expressions for the *n*th roots of unity, that is, complex numbers satisfying the equation $z^n = 1$. De Moivre never explicitly stated it in his work. However, his familiarity with it is clear from a related formula that he discovered in 1722, namely,

$$\cos \varphi = \tfrac{1}{2}(\cos n\varphi + i \sin n\varphi)^{1/n} + \tfrac{1}{2}(- .$$

De Moivre's chief works were *The Doctrine of Chances*, (1718), a key contribution to the early history of probability, the *Miscellanea Analytica*, (1730), in which he investigated infinite series, and *A Treatise of Annuities on Lives*, (1752), an application of probability to mortality statistics, and the creation of the theory of annuities. This remarkably original work laid the foundations of the mathematics of life insurance.

de Moivre was born in Vitry, near Paris and spent five years at a Protestant academy at Sedan. From 1682 to 1684, he studied logic at Saumur, then entered the Collège de Harcourt in Paris and took private mathematical lessons with Jacques Ozanam. De Moivre had the misfortune to be a Huguenot (Calvinist) at the time that Roman Catholic France revoked the Edict of Nantes and began persecuting French Protestants. De Moivre was imprisoned in Paris for a year and when he was released moved to England. He developed a friendship with Newton and Edmund Halley, which helped him to be elected

to the Royal Society in 1697. In 1710, he returned the favor as a member of the Royal Society's Commission to review the rival claims of Newton and Leibniz as the inventors of the calculus. Despite helping the Royal Society get the answer it wanted by supporting Newton, de Moivre later sought Leibniz's help in securing a university chair on the continent. However, nothing came of it and any hope of finding a mathematics chair in England was dashed because he was a foreigner. De Moivre studied Newton's *Principia* and became such an expert on it that in later years, when asked about some point or another in it, Newton would say, "Go to Mr. De Moivre; he knows these things better than I do."

In 1733 de Moivre derived what is now known as the normal distribution as a method for estimating discrete probabilities, in particular those involving the binomial distribution. He sought to determine the probability of the most frequent occurrences in a binomial distribution. De Moivre's concern was with games of chance, and his discovery showed the power of sampling to determine patterns in a population by examining only a few members or cases. Later Pierre de Laplace, motivated by observational science, discovered the means of various samples of n measurements are distributed approximately according to the normal curve. From these approximations Laplace was able only to state the high probability of sample means lying within a given range according to the normal distribution. This approximation of the probability of sums of binomial distribution values is now known as the de-Moivre-Laplace limit theorem. Prior to the discovery of this theorem, probability and statistics were treated as two separate entities. It was the first example of central limit theorems, most of which were derived by Pafnuty Tchebycheff and his students Andrei Markov and Aleksandr Lyapunov during the period 1880 to 1920. These theorems unified probability and statistics.

One of the most important examples of a continuous probability distribution, the normal distribution is often referred to as, "The Bell-Shaped Curve," the shape of its graph (Figure 5.11). An extremely wide

range of natural phenomena and are accurately described using the curve. The empirical rule of the normal distribution is that 68% of the area lies under the curve within one standard deviation of the center; 95% of the area under the curve lies within two standard deviations of the center; and 99% of the area under the curve lies within three standard deviations of the center. A standard deviation is the square root of the average of the squares of the deviations from the mean in a frequency distribution. De Moivre's *The Book of Chances* marked the first appearance of the bell-curve, although the origin of the curve is sometimes attributed to Gauss, who did the most important and fundamental work with the normal distribution; so much so that it is sometimes referred to as the Gaussian distribution.

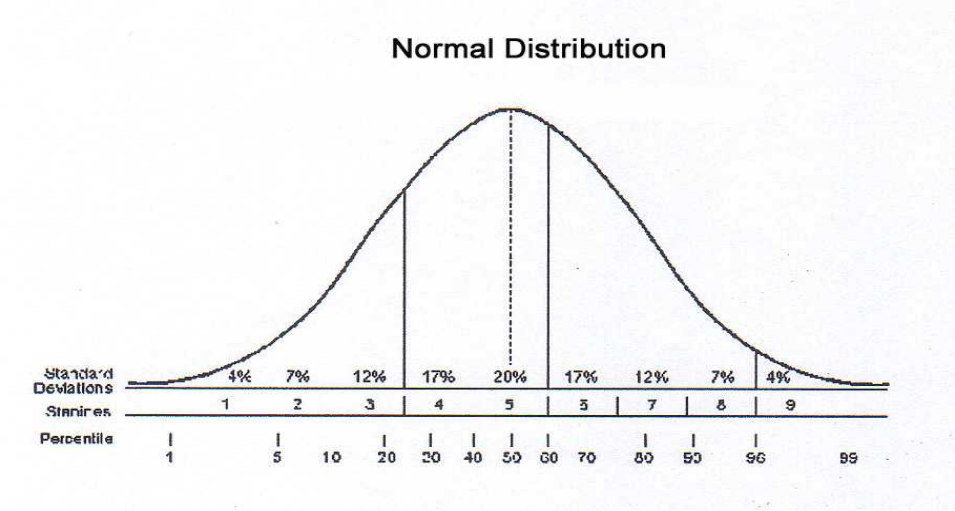

Figure 5.11

De Moivre remained at the poverty level all his life, reduced to working as a private tutor or as a consultant to gambling or insurance syndicates, and never obtaining a university position. At the end of his life, he eked out a living by solving problems of chance for gamblers as the resident statistician of Slaughter's Coffee House in London. He died blind and disillusioned, with his contributions to mathematics unrecognized. W.W. Rouse Ball wrote of de Moivre's final days:

"The Manner of de Moivre's death has a certain interest for psychologists. Shortly before it, he declared that it was necessary for him to sleep some ten minutes or a quarter of an hour longer

each day than the preceding one: the day after he had thus reached a total of something over twenty-three hours he slept up to the limit of twenty-four hours, and then died in his sleep."

Quotation of the Day: "And thus in all cases it will be found, that although Chance produces irregularities, still the Odds will be infinitely great, that in the process of Time, those irregularities will bear no proportion to the recurrence of that Order which natural results from Original Design…. Again, as it is thus demonstrable that there are, in the constitution of things, certain Laws according to which Events happen, it is no less evident from Observation, that these Laws serve to wise, useful and beneficial purposes, to preserve the steadfast Order of the Universe, to propagate the several species of Beings, and to furnish to the sentient Kind such degrees of happiness as are suited to their State." – Abraham de Moivre makes a case for his discovery of the normal distribution being a proof of the existence of a divine Creator.