Mohammad Abu'l-Wafa al'Buzjani

Mohammad Abu'l-Wafa al'Buzjani (June 10, 940 – July 15, 998), who was born at Buzjan in the Khorasan region known today as Iran, was one of the most prolific and well-known Islamic mathematicians and astronomers. He worked at the Baghdad observatory of his patron, the Caliph Sharaf al Daula. As one of the last translators and commentators on Greek works, he wrote many analyses, now lost, on the works of Euclid, Diophantus, and al-Khwarizmi, astronomical tables, a practical arithmetic, a simplified version of Ptolemy's *The Almagest*, and a book of applied geometry. Abu'l-Wafa's contributions to trigonometry were extensive. In his study of the orbit of the moon, he developed the knowledge of the *tangent* function and introduced a new method of constructing sine and tangent tables. His *Zijush Shamil* (consolidated tables) is distinguished by being accurate to eight decimal points, while those of Ptolemy were accurate only to three. Abu'l-Wafa is also credited with the discovery of the secant and cosecant functions, as well as relationships among all six of the trigonometric functions.

Abu'l-Wafa's contributions to geometry are found in his *Kitab al-Hindusa*, which consisted of solutions of a large number of geometrical problems on the fundamental construction of plane geometry with the opening of the compass. He constructed a square equivalent to other squares, regular polyhedra (based on the work of Pappus), a regular hectagon (a polygon with 100 equal sides), a parabola, and the geometrical solution of certain quartic equations. He also presented more complicated constructions such as Archimedean solids on the sphere.

Astronomy was of major interest to Arabic mathematicians, and their developments in trigonometry made it possible to construct much more accurate astronomical tables. Their interest was not a matter of mere intellectual curiosity. Islamic religious ritual required five daily prayers to be performed by the faithful at times regulated by the Sun's position. The prayers had to be said facing in the direction of the Kaaba in Mecca. Additionally, the Islamic calendar is based on lunar months, each month beginning at the first appearance of the lunar crescent after the new Moon. To ensure that the rules of Islam were faithfully followed, it was necessary to have precise knowledge of celestial and planetary motions, as well as terrestrial geography.

Abu'l-Wafa's proof of the "rule of four quantities" marks the transition in the study of triangular relations. Up to that time, triangular relations were studied using Menelaus' theorem, which required six quantities. Menelaus' proposition states that if, in the triangle ABC, points P_1, P_2 and P_3 are on the lines that contain AB, BC, CA, respectively, then P_1, P_2 and P_3 are collinear if and only if $(AP_1/ P_1B)(B P_2/P_2 C)(C P_3 / P_3 A = - 1$. Menelaus used spherical quadrilaterals to establish relationships. Abu'l-Wafa used triangles to reduce the number of quantities needed but he did not use angles, only the sides of triangles. He found the relationship between sides a and a', c and c' of two similar triangles to be $\sin a/ \sin a' = \sin c/ \sin c'$.

Not long after, Abu'l-Wafa was one of three astronomers who independently proved the law of sines: For a spherical triangle, the sides of are proportional to the sines of the opposite angles. The formulation made explicit the relationship among spherical triangles. A spherical triangle [Figure 6.1] is a portion of a spherical surface bounded by three arcs of great circles. The study of spherical triangles is spherical trigonometry and consists of finding unknown sides, angles and areas by the use of trigonometric functions of the plane angles that measure angles and sides of the triangles. There are spherical analogues of the cosine and sine formulas of plane trigonometry. Spherical trigonometry is vital to geodesy, navigation, and celestial mechanics. A position on the surface of the earth is located by means of latitude and longitude, where latitude is measured from the equator to the position. Similarly, the position of astronomical objects is determined by their declination, which is also measured from the equator. A navigational triangle [Figure 6.2] is determined by three points, two locations A and C and the North or South Pole, B.

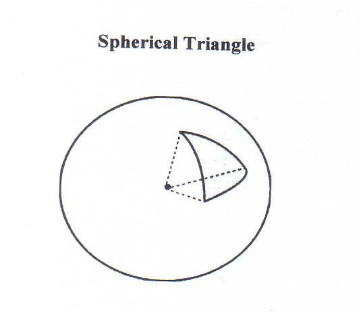

Figure 6.1

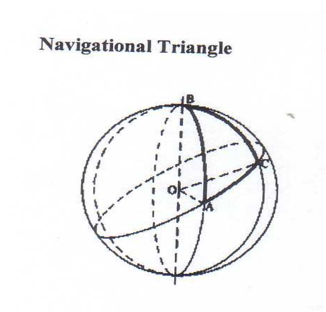

Figure 6.2

Abu'l-Wafa also wrote a complete textbook on Astronomy from a mathematical point of view with explicit solutions. He was the author of the *Book on what is necessary from the science of arithmetic for scribes and businessmen* and *a book on those geometric constructions which are necessary for craftsmen*. In his arithmetic, Abu'l-Wafa did not make use of the Indian numerals. All numbers were written in words and all calculations were done mentally. It wasn't that Abu'l-Wafa was unfamiliar with the Indian numerals; he was in fact quite an expert in their use. However, he knew the audience he was writing for, and businessmen preferred finger-reckoning arithmetic.

Quotation of the Day: "Perhaps to the student there is no part of elementary mathematics so repulsive as is spherical trigonometry." – Peter Guthrie Tait