

Karl Weierstrass

German mathematician **Karl Theodor Wilhelm Weierstrass**

(October 31, 1815 – February 19, 1897) has been acclaimed “the father of modern analysis” as well as the world’s foremost teacher of advanced mathematics. Surely Weierstrass was one of the greatest mathematicians of the nineteenth century. What is most remarkable is that he spent much of his early life and career isolated from other outstanding mathematicians while he taught



various subjects in secondary schools. It wasn’t until he was about forty that his genius was recognized and he secured a university position. He broadened the understanding of the notion of “function,” gave the modern δ, ϵ definition of limit, discovered and applied uniform convergence and gave a theory of irrational numbers as series of rationals. Previous to this time if mathematicians defined irrationals, they did so as limits of sequences of rationals. It was Georg Cantor, who made the reasonable observation that irrationals must already exist if they were limits of sequences.

Weierstrass was born at Ostenfelde in Westphalia, the eldest of four children of Wilhelm Weierstrass, a customs official. Wilhelm was an overbearing man who dominated the lives of his children. He continued to lecture Karl even when the latter was in his forties. Karl’s mother Theodora Vonderforst died in 1827. None of Wilhelm’s children ever married. In 1829 young Karl entered the Catholic Gymnasium in Paderborn. He excelled at the school while holding a part-time job as a bookkeeper to help the family finances. Although Weierstrass demonstrated considerable mathematical ability at the Gymnasium, he acceded to his father’s wishes and entered the University of Bonn in 1834 to study accounting, law, economics and finance. Bored with his courses, he seldom attended lectures; instead,

as reported by E.T. Bell, he “devoted his great bodily strength, his lightning dexterity and his keen mind almost exclusively to fencing and the mellow sociability that is induced by nightly and liberal indulgence in honest German beer.”

However, Weierstrass did study mathematics on his own. After four years he returned home to his enraged father without a degree. He next enrolled at the Theological and Philosophical Academy at Münster to prepare to teach in secondary schools. He became the student of Christof Gudermann, who gave Weierstrass personal attention. Gudermann shared with Weierstrass his enthusiasm for the idea that power series could be used as a rigorous basis for mathematical analysis. Karl received his degree in 1839, after writing a revolutionary essay on elliptic functions as part of his examination. In 1841 Weierstrass received his teacher’s certificate and began a one-year probation as a teacher at the Gymnasium in Münster.

For the next fifteen years, Weierstrass was employed as a secondary school teacher. He was required to teach a variety of courses besides mathematics, including botany, history, physics, German, geography, gymnastics and penmanship. During this period he was cut off from the mathematical community. With no mathematical colleagues to talk to and no mathematics libraries to visit, he was on his own in exploring mathematical ideas. He later described this period of isolation as one of “unending dreariness and boredom.” During the days he was a dedicated teacher devoted to the well-being of his students, but during the nights he pursued the study of mathematics; in particular the work of Niels Abel.

Weierstrass was determined to continue Abel’s pioneering work and for years did remarkable research on Abelian functions. He did not discuss his work with anyone nor submit anything for publication. Starting about 1850, Weierstrass suffered from extreme vertigo with spells lasting for about an hour and ending with violent vomiting. The attacks occurred frequently for the next 12 years, making his already demanding work load and his independent pursuit of mathematics all the more difficult. In 1861 he

collapsed while lecturing and was unable to return to the classroom for over a year. From then on, he lectured while seated, leaving any necessary blackboard work to a student assistant.

Weierstrass' first publication on Abelian functions appeared in the Braunschweig school prospectus (1848-49) and went unnoticed. Then in 1854 Weierstrass published a general theory of Abelian integrals and Abelian functions in *Crelle's Journal*, early volumes of which were filled with Abel's papers. This memoir finally brought Weierstrass to the attention of the mathematical world. In the paper he gave a preliminary description of methods involving the representation of Abelian functions as convergent power series. The University of Königsberg conferred an honorary doctor's degree on him on March 31, 1854. The next year Weierstrass applied for the position vacated by Kummer at the University of Breslau, but did not get the position. After being promoted to senior lecturer at Braunschweig, he took a year's leave of absence with pay to pursue advanced mathematical study. He was determined never to return to secondary school teaching and settled for a chair with the Industry Institute in Berlin. Finally, in 1856, he realized his dream when he was offered a professorship at the University of Berlin where he remained the rest of his life. After he stopped lecturing at the university, he gave private lessons to particularly talented students. His relationship with Sofia Kovalevskaya is detailed in her entry. He was so devastated by her death at age 41 that he burned everything he owned that reminded him of her.

Weierstrass published little, but was famous for the lectures in which he gave a systematic account of mathematical analysis based on extremely careful reasoning that became known as "Weierstrassian rigor." When he published his *Werke* in the 1890s, he was more concerned with presenting his method of developing function theory than establishing the priority of his results, many of which had been published by others in the meantime. Weierstrass was a methodical man who distrusted flashes of intuition, preferring to painstakingly develop his mathematical ideas. He proved what is known as the Weierstrass Approximation Theorem, that is, "Any continuous function of a bounded interval can be

uniformly approximated by polynomial functions.”

Weierstrass shocked some in the mathematical community by inventing new functions that were so “bizarre” that Poincaré described them as “deplorable evil.” For instance, it previously had been supposed, and even “proven” in some textbooks that all continuous functions except for some isolated cases are differentiable. This belief arose from limiting functions to those defined by algebraic formulas. Weierstrass found counterexamples, including a continuous function that is not differentiable at any point. This opened a whole new way of thinking about functions. He clarified the notions of the fundamental concepts of analysis, giving function, derivative, limit, differentiability, and convergence their first rigorous definitions. His example of a continuous function that is nowhere differentiable is:

$$f(x) = \sum_{n=1}^{\infty} b^n \cos(a^n x) \text{ where } b < 1 \text{ and } ab > 1 + \frac{3\pi}{2}$$

The presence of Weierstrass and his colleagues Leopold Kronecker and Ernst Kummer gave the University of Berlin a reputation as the leading university at which to study mathematics. For twenty years Kronecker and Weierstrass were close friends, sharing many fruitful discussions. A rupture was prompted by the work of Georg Cantor. Weierstrass was among the first to recognize the importance of Cantor’s work with transfinite numbers and countability. Before Cantor, mathematicians accepted the notion of a sequence “tending to infinity,” but did not accept the notion of infinity as a number. Cantor’s great achievement was to introduce just such a concept. Kronecker, who did not accept non-constructive reasoning, denounced the work and set himself the task “of investigating the error of every conclusion used in the so-called present method of analysis.” Kronecker was not satisfied with making Cantor’s life miserable and sabotaging his career, but he also lashed out at Weierstrass. It was the

latter's work on limits and convergent sequences of rationals that led his student Cantor to his revolutionary theories. The rift was so bad that, in 1885, Weierstrass decided to leave Berlin for Switzerland. He changed his mind when he realized that if he left, Kronecker would have to endorse his successor. He stayed and when Kronecker died in 1891 the path was cleared for Weierstrass to be succeeded by his former student Hermann Schwartz. As a teacher, Weierstrass attracted large audiences, often in excess of 200. In 1875 he became a knight of the order "Pour le Mérite" in the category of Arts and Sciences, which was the highest honor that the newly unified German nation could bestow. Because of failing health, he retired at age 75, spending the last few years of his life confined to a wheelchair, dying from pneumonia at age 82.

Quotation of the Day: "It is true that a mathematician who is not also something of a poet will never be a perfect mathematician." – Karl Weierstrass