John Wallis

Among the leading English mathematicians contemporary to Isaac Newton was **John Wallis** (November 23, 1616 – October 28, 1703). He deserves at least partial credit for the development of modern calculus. Mathematician, logician, grammarian, skilled linguist, and Divine, Wallis wrote brilliantly and voluminously, demonstrating a genius for mathematics. His work would be more esteemed had it not been overshadowed by that of Newton. Wallis was one of the first to recognize the significance of the generalization of exponents to include negative and fractional as well as positive and integral number. He proved the laws of exponents, which led to defining x^0, x^{-1}, x^{-2}, … to mean 1, $1/x$, $1/x^2$ respectively; $x^{1/2}$ to represent the square root of x, $x^{2/3}$ to mean the cube root of x^2, and generally $x^{-n} = 1/x^n$ (where $n \neq 0$), and $x^{p/q}$ = the qth root of x^p ($q \neq 0$). He was one of the first to discuss conics as curves of second-degree polynomials rather than as sections of a cone. His most important work was *Arithmetica Infinitorum* (1665, *The Arithmetic of Infinitesimals*), which stimulated Newton's work on calculus and the binomial theorem. In the *Arithmetica* Wallis was among the first to accept negative numbers and to recognize them as roots of equations. However, he wrongly thought negative numbers to be not less than zero, but greater than ∞, the symbol he is credited with introducing to represent infinity.

Born at Ashford in Kent, our subject was the third of five children from the union of the Reverend John Wallis and his second wife, Joanna Chapman. He attended James Movat's grammar school at Yenterden and then the school of Martin Holbreach at Felsted, where he was instructed in Latin, Greek and

Hebrew. At that time mathematics was not considered important enough to be part of the curriculum, so Wallis did not encounter the subject until the Christmas holidays of 1631 when his elder brother taught him arithmetic. Wallis was captivated and read every mathematics book he could find. At Emmanuel College, Cambridge Wallis studied ethics, metaphysics, geography, astronomy, medicine and anatomy, but no mathematics. He received his M.A. in 1640, the same year he was ordained by the Bishop of Winchester. For the next few years he held positions as a chaplain in Yorkshire, Essex and in London.

The course of Wallis' life was changed during the English Civil War when he put his almost miraculous skills to work deciphering coded Royalist messages for the Parliamentarians, and encrypting messages for the Puritan army. His loyalty was rewarded when he was made rector of St. Gabriel Church in London and in 1644 secretary to the Westminster Assembly of Divines. That same year he married Susanna Glyde. Despite his adherence to the Puritan party, Wallis joined the moderate Presbyterians in signing the remonstrance against the execution of Charles I. Oliver Cromwell appointed Wallis the Savilian professor of geometry at Oxford over the objections of the university community who regarded a royalist to be the rightful appointment. After the Restoration of the monarchy Charles II confirmed Wallis in his position (which he held from 1649 to his death), appointed him royal chaplain, and nominated him to be a member of the committee formed to revise the prayer book.

Probably no other scholar of the period became embroiled in more heated intellectual disputes than did the remarkably quarrelsome Wallis. One of the most bitter public disputes was with Thomas Hobbes, which for some 25 years generated an avalanche of polemical treatises not only from the two combatants, but also from other British and continental mathematicians. Wallis maintained that he would not have taken the time to refute Hobbes' mathematical writings had it not been that his reputation in philosophy was a danger to religion. Wallis's real aim in discrediting Hobbes' mathematics was to undermine the epistemological basis of Hobbes' materialist philosophy, which maintained that human action was motivated entirely by selfish concerns, most notably fear of death.

Among Wallis' many mathematical works was his treatise *Tractatus de sectionibus conics* (1655), in which he defined conic sections analytically, making Descartes' obscure methods more comprehensible. Wallis used Cavalieri's method of indivisibles to find the areas between curves of the form $y = x^n$, the x-axis and the vertical line $x = h$ for some $h > 0$. This is equivalent to evaluating the definite integral

$$\int_0^h x^n \, dx = h^{n+1}/(n+1) \text{ where } n \neq 0.$$

Wallis' *De Algebra Tractatus; Historicus & Practicus* (1685, *Algebra: History and Practice*) was the first serious effort to write a history of mathematics. He also wrote on proportion, mechanics, grammar, logic, decipherment, theology, and the teaching of the deaf, and edited ancient mathematical manuscripts. The meetings of mathematicians and astronomers, which he hosted at his quarters at Gresham College, London, led to the establishment of the Royal Society of London for the Promotion of Natural Knowledge, which Charles II formal chartered in 1662. Wallis demonstrated his deep respect for tradition, explaining in his autobiography: "I made it my business to examine things to the bottom; and reduce effects to their first principles and original causes. Thereby the better to understand the true ground of what hath been delivered to us from the Ancients, and to make further improvements of it."

Even though he was aware of the inaccuracies of the Julian calendar, especially in determining the yearly date for Easter, Wallis opposed the introduction of the Gregorian calendar. Unwilling to adopt the creation of Popish Councils, he offered and alternate means of finding Easter without changing the civil calendar, but couldn't win its approval. Wallis found a method for calculating the value of π by finding the area under a quadrant of a unit circle (a circle with radius one) (Figure 10.18). Using the formula $A = \pi r^2$, the area of a unit circle is π, so the area of the quadrant of the unit circle is $\pi/4$. Through a long series of interpolations and inductions, he derived what is now known as Wallis' Product:

$$\pi/2 = 2/1 \cdot 2/3 \cdot 4/3 \cdot 4/5 \cdot 6/5 \cdot 6/7 \cdot \ldots\ 2k/(2k\text{-}1) \cdot 2k/(2k+1) \cdot \ldots = \prod_{k=1}^{\infty} (2k)^2/(2k-1)(2k+1)$$

The advantage of the formula is it doesn't involve irrational numbers and the calculations are relatively easy. The disadvantage is that the approximation is not very accurate. For example, with $k = 30$, $\pi \approx 3.11595$.

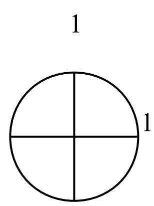

Figure 10.16

Throughout his life, Wallis fought for the priority claims of Englishmen against foreigners. It has been said that because of his xenophobia, he offended every foreign mathematician he ever met. When it seemed possible that Newton's priority in the question of the calculus might be endangered by the claims for Leibniz, Wallis eagerly jumped into the fray. In 1695 he published his *Mathematical Works*, with Volume II his *Algebra* with an essay on fluxional calculus. In it Wallis claimed that he learned the method in letters from Newton in 1676. From this those who wished to do so could imply that it was possible that Newton had communicated his methods of fluxions to Leibniz in 1676 prior to the latter's publication of his calculus.

In his *Philosophers at War*, Rupert Hall writes, “Inadvertently, therefore, Wallis was beginning a process of public deception.” It’s not too great a stretch to cast Wallis in the part of Iago to Newton’s Othello and Leibniz’s Desdemona. Newton and Leibniz’s relationship was cordial and mutually respectful, until Wallis stirred the pot by figuratively whispering in Newton’s ear that in Holland: “Your notions are known as Leibniz’s *Calculus Differentialis*,” and urged his friend to take steps to protect his reputation. Thus, Wallis poisoned the atmosphere, leading to a bitter jingoistic dispute that damaged British mathematics.

Quotation of the Day: “Wallis’ own role in the slow warming of the calculus dispute had been to act as an uncritical mouthpiece for Newton … Wallis was there (*Algebra*) writing – as so many Newtonians were to write – under Newton’s instructions.” – Rupert Hall