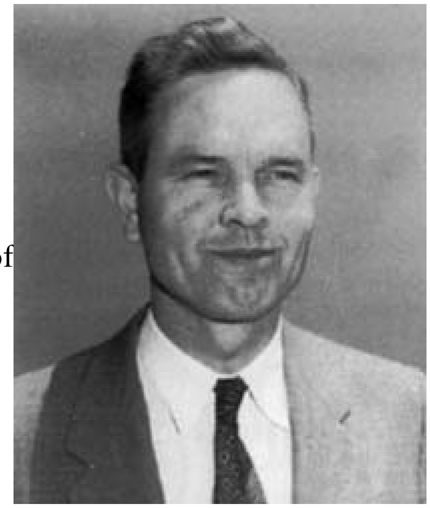
Norman Steenrod



Norman Earl Steenrod (April 22, 1910 – October 14, 1971), among the leading topologists of the 20th century, collaborated with Samuel Eilenberg on writing *Foundations of Algebraic Topology* (1952), one of the primary sources in the field. They developed axioms for analyzing homology theory, one of the foundation stones and the oldest part of algebraic topology. Homology theory is a transition from topology to algebra, and it is this transition that Eilenberg and Steenrod axiomatized. Historically, Henri Poincaré was the first to use the term *homology* in a topological sense, but only within an intuitively based setting. Roughly speaking a homology theory assigns groups to topological spaces and structure preserving mappings to continuous functions of one space to another. A homology theory is an *algebraic image* of topology. As Eilenberg and Steenrod said in their preface, the *domain* of a homology theory is the topologist's field of study, while its *range* is that of the algebraist's.

Steenrod's parents were teachers who didn't have any particular interest in mathematics. His father shared his astronomical hobby with his son and his mother helped him develop a lifelong interest in music. He was born in Dayton, Ohio, where he finished the customary twelve years of elementary and secondary school in nine. As he was felt to be too young to go off to college, and in need of funds to help pay for further education, he spent two years working as a tool designer. In 1927 he entered the University of Miami at Oxford, Ohio, but was forced to interrupt his college education several times to earn money for his expenses. He transferred to the University of Michigan, where he studied physics and philosophy, graduating in 1932. He only took one mathematics course at Michigan, but it proved to be instrumental in deciding the direction of his life. It was a topology course offered by Raymond L.

Wilder, who gave Steenrod his first research problem.

In the 1930s the nation was still reeling from the effects of the Great Depression of 1929. By late 1932 stock values had fallen to only about 20% of their previous value, and by the next year 11,000 of the nation's 25,000 banks had failed. There was a much-reduced level of demand for products and as a consequence unemployment reached 25 to 30%. It wasn't until 1939, with World War II beginning that it could be said the Great Depression was over. It was not a good time for a young man to pursue his education and a career. He spent the year 1932-33 working back home in Dayton. He concentrated on the topology problem Wilder had given him and in 1933 published his first research paper. The quality of his work was obvious and he had offers of fellowships from Harvard, Princeton, and Duke from which to choose. He chose Harvard, but before moving to Cambridge, he spent time working as a die designer at a Chevrolet plant in Flint, Michigan. He earned a master's degree in 1934 and again found he had several further educational options. He was about to accept a fellowship from the University of Michigan to be reunited with Wilder when he discovered the latter was moving to Princeton. That was convenient since Princeton also offered Steenrod a scholarship. Two years later, he was awarded a doctorate from Princeton for a thesis *Universal Homology Groups*, supervised by Solomon Lefschetz.

After receiving his Ph.D, Steenrod remained at Princeton as an instructor, and then accepted a position with the University of Chicago where he stayed from 1939-1942, before returning to Michigan. In 1947 Steenrod joined the mathematics faculty of Princeton University where he remained for the rest of his career. He played a crucial role in the development of fibre bundles. The structure of a fibre bundle consists of a total space E, a base space B, the fibre F, a projection map p: $E \rightarrow B$, such that$p^{-1}(x)$ is homeomorphic (topologically equivalent) to the fibre F. To illustrate the concept recall that a Möbius strip can be thought of as a line segment twisting as it moves around a circle. Think of the Möbius strip as the total space, the base is a circle, and the line segment as the fibre. Besides a line segment, different other shapes can be moved round a circle, with various amounts of twisting. The results are *fibre bundles*. The *fibre* is the form of the shape that is moved around the circle. Besides these two ideas the additional data needed to describe a fibre bundle is the amount of *twist* in the fibre as it moves around the base. Fibre bundles play an integral role in differential geometry, as well as modern physics. Steenrod's *The Topology of Fibre Bundles* (1951) is a classic and a work of great utility to mathematicians and theoretical physicists alike. Herbert Seifert introduced fibre spaces in a doctoral thesis, *Topology of 3-dimensional fibred spaces* in 1932. The definition of a fibre space differed from country to country until Jean-Pierre Serre revised it in his doctoral thesis, after which it developed into a mature subject, greatly enhanced by Steenrod's influence.

According to Colin McLarty, Steenrod spent years trying to axiomatize homology. When Eilenberg and Saunders MacLane collaborated in creating *category theory* in papers of 1942 and 1945, Steenrod saw that this was the key to solving his problem. Category *theory* is a relatively new field of mathematics that provides a universal framework for discussing structures and systems of structures, such as those found in algebra and geometry. It offers means of seeing how structures of different kinds are related to one another, and it is considered as an alternative to set theory as a foundation of mathematics. Although general category theory has attracted considerable interest in many fields, it has been most fruitful in group theory and algebraic topology, as an important part of the transition from the intuitive and geometric concept homology to homology theory, an axiomatic approach. Steenrod dubbed category theory "great abstract nonsense." Serge Lang popularized the phrase in his *Algebra* (1965). In the index there are several page references to "abstract nonsense," and when these are looked up, one finds various one-line proofs that begin with "By abstract nonsense, *such and such is so*."

When I completed my Ph.D., I gave a copy of my thesis on a topic related to fibre spaces to my businessman father to examine. Before opening it he said, "I may not understand all of this." I responded, “I’ll give you a thousand dollars if you understand anything in it.” This was not a knock of my father’s intelligence. It’s just the work required a considerable amount of knowledge of mathematics, especially in the field of algebraic topology, to understand the definitions, let alone the proofs of the theorems. This is how it is with doctoral dissertations in mathematics. As Steenrod advised incoming graduate students, “In your undergraduate studies the mathematics that you have read has been primarily in textbooks. But now you are ready to read original articles – articles are living mathematics, and textbooks are dead mathematics. You should read original articles, even if they’re harder and not so well written.” These mathematical articles and theses are not written for the general public, but rather for a select number of other mathematicians, who with a little effort will understand the results and the significance of the material. Only after extended periods of dissemination, refining, simplifying and relating to other mathematical ideas will original mathematical work find its way into textbooks [and perhaps not even then] where greater numbers of people will understand it. Steenrod aside, calling such mathematics dead is rather harsh. Let’s merely say that it is no longer fresh.

Quotation of the Day: “Steenrod reminded me of Rodin’s thinker (not in physique, but rather in the way in which he reveled in slow reflection). He made it clear that quickness was not essential for doing good mathematics; the important problems in mathematics, or at least the ones he valued, required having an appropriate view of the whole, and such overviews are gotten only after considerable experience, and long deliberation.” - Barry Mazur, a former student of Steenrod.