Wacław Sierpiński

In the 18th century Poland was divided among Prussia, Russia, and Austria, and did not exist as a nation. From 1863 until World War I, it was a province of Russia, but after the war, the victors established an independent Poland. In the 1920s, a group of patriotic Polish mathematicians under the forceful leadership of **Wacław Sierpiński** (March 14, 1882 – October 21, 1969) decided that rather than try to re-build comprehensive university programs in several areas of research, they would establish their independence from the leading mathematical centers of Europe. To do so, they built an abstract Polish school of mathematics, concentrating on set theory and topology, fields not getting much play in European intellectual capitals. Sierpiński was a prolific author, publishing an incredible 50 books and more than 700 papers on set theory, topology, number theory, function theory, and logic. He also made important contributions to the field of fractals.

Sierpiński was born in Warsaw, the son of a renowned physician. The Russians, who occupied Poland, forced their own culture and language on the Poles. Schools and universities in Poland were really Russian institutions. The teachers were all Russians and the instruction and lectures were in Russian. They reasoned that if the Poles were kept illiterate they were unlikely to shake off the yoke of their conquerors. Even in his youth Sierpiński gave evidence of his public spirit when, together with a few friends from his secondary school, he organized secret courses for boys who could not afford to go to school. In 1899 he entered the Department of Mathematics and Physics at the University of Warsaw, whose official name was the Czar's University. He won a gold medal for an essay he wrote on the contributions to number theory of his Russian teacher Georgy F. Voronoy in 1903. That same year, Sierpiński graduated from the University with the degree of "candidate of sciences" and went on to teach mathematics and physics at a Warsaw girl's school. He left teaching to pursue a doctorate at the Jagiellonian University in Krakow, which he received in 1908.

On his return to Warsaw Sierpiński was given permission to teach mathematics in the secondary schools in Polish. He gave "Scientific Courses," offered by an unofficial and illegal Polish University. In 1908 Sierpiński was elected a member of the newly established Warsaw Scientific Society and appointed Assistant Professor at the Jan Casimir University in Lvov [Lwow]. During this period Sierpiński's mathematical interests evolved from number theory, in which he had achieved excellent results, to set theory. His *An Outline of Set Theory* (1912), based on a textbook he wrote in 1909, is one of the first synthetic formulations of the theory.

At the outbreak of WWI, the Russians interned Sierpiński in Moscow. At the end of hostilities, he returned to Poland to become a professor at the newly reborn Polish University of Warsaw. In the years that followed he was one of the main creators of Poland's famous school of set theorists and topologists. To have a journal where their work could be published, in 1920 Sierpiński and Zygmunt Janiszewski founded the still important journal *Fundamenta Mathematica*. Janiszewski was to be editor-in-chief, but died before the first volume appeared in print. Sierpiński assumed the post of editor and kept it for many decades.

In 1915, Sierpiński created several self-similar patterns and the functions that generate them. He thought of what Benoit Mandelbrot named the *Sierpiński Gasket*. It is a fractal obtained by beginning with an equilateral triangle in the plane with its interior. It is then divided into four smaller congruent equilateral triangles, of which the middle one is removed, producing a triangular hole. Each of the three remaining equilateral triangles is divided into four smaller equilateral triangles, of which the interior of the middle triangle is removed (Figure 10.11) (that is, do not remove the boundary). This process is repeated over and over again. The actual fractal is what would be obtained after an infinite number of iterations. The limiting figure of this process is the *Sierpiński Gasket*.

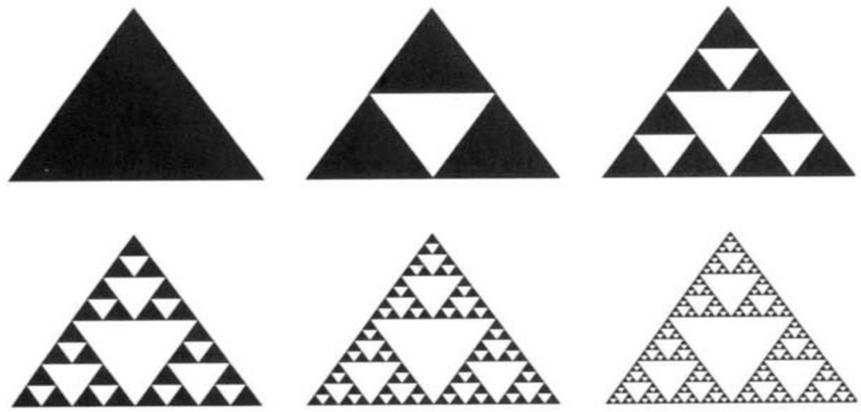

Figure 10.11

Of particular interest are the area and the length of the boundary of the gasket. If the area of the original triangle is 1, the first iteration removes 1/4 of the area, the second iteration removes a further 3/16, the third iteration removes an addition 9/64, and the total area removed after the nth iteration is given by:

$$A_n = \tfrac{1}{3} \sum_{i=1}^{n} (\tfrac{3}{4})^i, \text{ the limit as } n \rightarrow \infty \text{ is } A_\infty = 1 \text{ that is, the gasket has no area.}$$

If the perimeter of the original triangle is 1, then after the first iteration the perimeter increases by 1/2, after the second iteration it increases by 3/4, and after the nth iteration, the perimeter is given by:

$$P_n = 1 + \tfrac{1}{3} \sum_{i=1}^{n} (3/2)^i \text{ , the limit as } n \rightarrow \infty \text{ is } P_\infty = \infty,$$

that is, the gasket's boundary is of infinite length. A Sierpiński gasket constructed from resistors forms a simple network that can be used to study electrical conductivity and thermodynamic properties.

The *Sierpiński Carpet* is generated by a close analogy to the Sierpiński Gasket. Start with a square in the plane with its interior. Divide this into 9 smaller congruent squares. Remove the interior of the center square (that is, do not remove the boundary). Next subdivide each of the eight remaining solid squares into 9 congruent squares and remove the center square from each (Figure 10.12). Continuing this process results in a decreasing sequence of sets. The Sierpiński Carpet is the intersection of all the sets in the sequence, that is, it is the limiting figure as the process is repeated infinitely. It is an example of a curve that is simultaneously a Jordan curve and a Cantor curve. Sierpiński acknowledged that the curve was actually invented by his PhD student Stefan Mazurkiewicz, whose 1913 thesis was on curves filling a square.

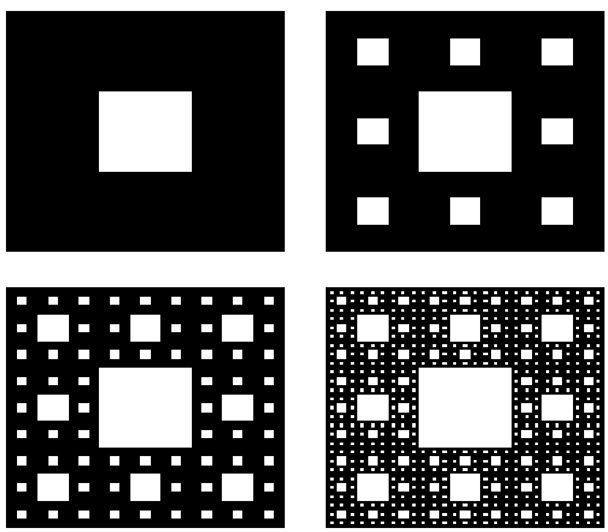

Figure 10.12

The *Sierpinski Sponge*, which more accurately is known as the *Menger sponge*, named for its inventor Austrian mathematician Karl Menger (1902 – 1985), is a fractal solid constructed by subdividing a cube into 27 smaller cubes (Figure 10.13). It is the three-dimensional analog of the Sierpiński carpet.

*T*he central cube and it six nearest neighbors are removed, leaving only the cubes on edges and corners. Each of the remaining twenty small cubes is processed in the same way. The Sierpiński or Menger Sponge is the limiting figure in this process. The topological equivalent of all one-dimensional objects can be found in the fractal. The area of the sponge as the iterations approaches infinity approaches zero, while its perimeter approaches infinity. The sponge is often used as a challenge to the computer graphics community.

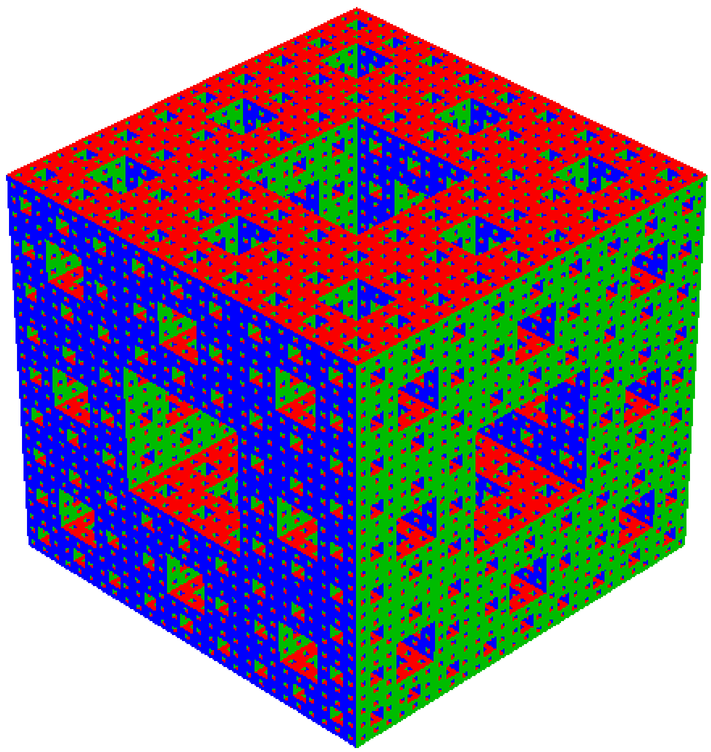

Figure 10.13

The two decades between the world wars saw the greatest flourishing of Sierpiński's talents, illustrated by the number of his publications, and the number of his students, which included most of the top young Polish mathematicians of two generations. With the outbreak of WWII and the German occupation, the Poles found themselves back where they were when the Russians ruled them. Once again the conquerors attempted to obliterate Polish culture, closing schools and universities, interning and murdering teachers. During this period, Sierpiński, whose official work was as a clerk in the council offices, taught in underground universities, conducting classes with a small group of students as the conspiratorial conditions permitted. His home, with the help of his wife, became the meeting place of trusted friends, students and colleagues, where news was exchanged and means found to help those most endangered.

Sierpiński kept up his spirit by continuing his research. He wrote numerous articles, textbooks and revisions of books, which he managed to send to Italy for publication. Each of his papers ended with the words: *The proofs of these theorems will appear in the publication of the Fundamenta Mathematica*, which everyone understood to mean "Poland will survive." After the Warsaw uprising in 1944 the Nazis burned Sierpiński's house destroying his library and personal letters. He was moved to the environs of Krakow. After the liberation of that town he lectured for a short time at the Jagiellonian University before returning to the reopened University of Warsaw, where he stayed until his retirement in 1960. He continued to give seminars on number theory at the Polish Academy of Sciences. He also served as editor-in-chief of the journal *Acta Arithmetica* that he founded in 1958.

Sierpiński's influence on his students in Poland and abroad, where he often traveled, was decisive. He inspired them with his ideas, helped them overcome the difficulties they encountered and influenced them with his own style. He was teacher, colleague, and friend to them, unconcerned with rank. In his seminars all that mattered were the contributions of the participants. He was a productive and creative mathematician who appreciated other creative and productive people. At nearly eighty Sierpiński was still an enthusiastic teacher. At a social gathering at a university he was visiting, a member of the assemblage related an anecdote about Sierpiński. For several years his wife had been concerned by his absent-mindedness. Once, when they were moving to a new home, while they were on the street waiting with all their things, she told him, "Now, you stand there and watch our ten trunks, while I go find a taxi." She went off and left him alone with their trunks. Some minutes later she returned with the cab. He said to her, "My dear, I thought you said there were ten trunks, but I've only counted nine." She insisted, "No, there are ten." He replied, "No, just count them: 0, 1, 2, 3, …" Asked if it was a true story, Sierpiński's eyes twinkled, he smiled, and said, "Well, perhaps."

Quotation: "… more than half of the mathematicians who lectured in our academic schools were killed. … In addition to the lamented personal losses Polish mathematics suffered because of the German barbarity during the war, it also suffered material losses. They burned down Warsaw University Library, which contained several thousand volumes, magazines, mathematical books and thousands of reprints of mathematical works by different authors. Nearly all the editions of *Fundametica Mathematica* and ten volumes of *Mathematical Monograph* were completely burned. Private libraries of all the four professors of mathematics from Warsaw University and also quite a number of manuscripts of their works and handbooks written during the war were burnt too." – Wacław Sierpiński