Kurt Reidemeister

Magicians thrill audiences with a trick in which the illusionist is tied with ropes, placed in a bag, and the bag put in a locked trunk. An assistant stands on the trunk holding a sheet that covers her and the trunk, begins a countdown, and suddenly disappears, the magician reappears on top of the trunk finishing the count. When opened the trunk reveals the girl is in the bag tied with the ropes. One thing that must be said for such magicians is that they have a fine working knowledge of knots, and how to quickly untie them. German mathematician **Kurt Reidemeister** (October 13, 1893 – July 8, 1971) discovered a great deal about knots. In 1926 he published *Knoten und gruppen* (*Knots and Groups*), the first monograph in a field just then beginning to emerge as an autonomous mathematical theory.

Reidemeister was born in Brunswick and completed his doctorate with a dissertation on algebraic number theory in 1921 at Göttingen. He was appointed an assistant professor of geometry at the University of Vienna in 1923, where his colleague Wilhelm Wirtinger interested him in knot theory. Wirtinger showed Reidemeister how to compute the fundamental group of a knot from its projection. This led to his 1926 book and in 1932 he brought out his *Knotentheorie* (*Knot Theory*) in which he sketched a proof that two knots are equivalent (that is they can be continuously deformed into each other) if and only if their projections are related by a finite sequence of operations, now known as the Reidemeister moves.

An early interest in knots is found in Greek legends. In Greek mythology, an oracle told the Phrygians that the next person to ride up to the temple of Zeus driving an oxcart would become their monarch.

Gordius, a poor stranger, innocently appeared at the shrine riding in a wagon drawn by an ox and was made king. In gratitude he dedicated his oxcart to Zeus and tied its shaft with an intricate knot of cornel bark. This became known as the Gordian knot. Another oracle foretold that the one who untied the knot would rule all Asia. The knot remained tied until Alexander the Great arrived. To save time he sliced through the knot with his sword. From that time on "cutting the Gordian knot" meant solving a difficult problem.

A mathematical knot has no loose ends. It is a simple closed curve that winds through itself in 3-space and reconnects with its tail to form a closed loop. If a particular tangled loop doesn't really have a knot in it and the loop can be unraveled and smoothed out to a simple closed curve (the topological equivalent of a circle), the configuration is called an unknot or a trivial knot. Lord Kelvin became interested in knots as he sought to develop a theory of atoms that would explain their stability, their variety, and their vibrational properties. He noticed the stability and vibrational properties of the smoke rings of his friend P.G. Tait. This led Kelvin to think of atoms as knots of swirling vortices in the ether.

Tait prepared a list of knots to see if there was any relation with the elements of the periodic table. Vortex theory soon disappeared but Tait's studies were important to the classification of knots according to their minimal crossing number, a knot invariant that is seen in a projection of a knot onto a plane. Today knot theory has grown into a subject with many applications, for instance in DNA replication and recombination, molecular chemistry, particle physics, computer science, and areas of statistical mechanics.

Reidemeister proved that if one has two different representations (or projections) of the same knot, one could be made to look like the other by using just three simple types of moves. They are: twisting or untwisting, that is, the adding or removing a crossing [Figure 7.1]; adding or removing two crossings, laying one strand over another [Figure 7.2]; and sliding a strand from one side of a crossing to the other [Figure 7.3]. In classifying knots, they are first simplified using Reidemeister's moves to get as few crossings in the knot as possible.

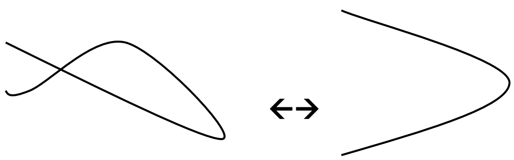

Figure 7.1

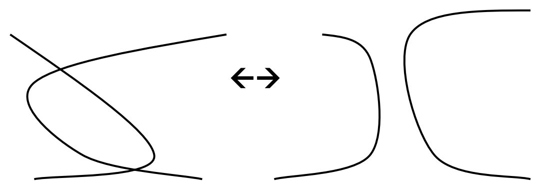

Figure 7.2

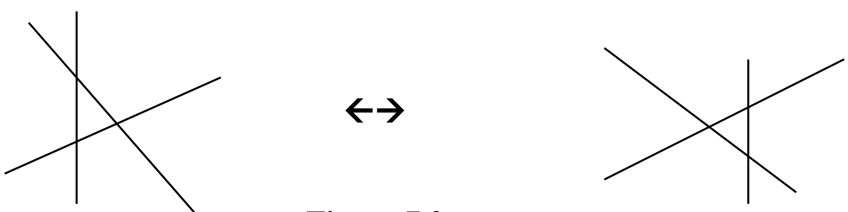

Figure 7.3

In 1927, Reidemeister was appointed to a chair at Königsberg. The Nazis, whom he strongly opposed, suspended him from this post six years later. He was classified as "politically unsound", because of his disapproving comments on disturbances caused by Nazi students. Later he filled the chair at the University of Marburg, a decidedly less prestigious post.

Besides knot theory, Reidemeister worked in the foundations of geometry and wrote *Einführung in die kombinatorische Topologie* (1932), an important combinatorial topology book. Reidemeister also was interested in philosophy and the foundations of mathematics. While at Vienna, in 1924, he learned of Ludwig Wittgenstein's *Tractatus Logico-Philosophicus*. Reidemeister led a group of mathematicians including Moritz Schlick, Hans Hahn, Otto Neurath, Victor Craft, Felix Kaufmann, and Rudolf Carnap who assigned themselves the task of clarifying the meanings of the basic concepts of philosophy. They pledged themselves not to become bogged down attempting to answer anything that could not be answered by fact or experience. The group broke up around 1939 having achieved little of their goal.

Quotation of the Day: In “Loopy Solution Brings Infinite Relief,” *Science* February 9, 2001, Charles Seife wrote about Jeff Lagarias and Joel Haas’s discovery of an upper bound on the number of Reidemeister moves required to remove all the crossings from the projection of a topologically unknotted curve.

“Finite numbers … can still be ridiculously large. All Lagarias and Haas guarantee is that if a knot crosses itself n times, you can untangle it in no more than $2^{100,000,000,000n}$ Reidemeister moves…. If every atom in the universe were performing a googol googol googol Reidemeister moves a second from the beginning of the universe to the end of the universe, that wouldn’t even approach the number you need to guarantee unknotting a single twist in a rubber band.”