JOSEPH LIOUVILLE

Joseph Liouville (March 24, 1809 – September 8, 1882) was one of the leading French mathematicians in the generation between Évariste Galois and Charles Hermite. He was born at Saint-Omer, Pas-de-Calais, the son of an army captain in Napoleon's army. Liouville's early education was under the supervision of his uncle with whom he lived while his father was away on military campaigns. He attended the Collège St. Louis in Paris, where he studied advanced mathematics. In 1825 he entered the École Polytechnique and was taught by André Marie Ampère and François Arago. After graduating from the Polytechnique in 1827, Liouville entered the École des Ponts et Chaussées from which he resigned in 1830. The next year he was appointed to his first academic position and then spent four years as an assistant to Claude Mathieu at the Polytechnique. In 1836 he founded the *Journal de Mathématiques Pures et Appliqées*, which is often referred to as *Liouville's Journal*. By this time he had gained an international reputation for papers that had been published in *Crelle's Journal.* In 1838 Liouville was appointed Professor of Analysis and Mechanics at the Polytechnique and two years later he was elected to the Bureau des Longitudes.

During the revolutions of 1848, Liouville was elected as a moderate republican to the constituent assembly, but was not reelected the following year. The sortie into politics altered his personality, leaving him hostile even towards his old friends. Despite health problems, his mathematical output, which had been interrupted by his political venture, picked up. His interests were wide, ranging from mathematical physics to pure mathematics. Liouville is also remembered for his work with transcendental numbers. He studied numbers, now called "Liouville numbers," that is, a real number x with the property that, for any integer n, there is a rational number p/q such that $q > 1$ and $|x - p/q| <$

$1/q^n$. Liouville showed that not only are these numbers irrational, they are always transcendental. Thus he was the first to prove the existence of transcendental numbers and also that there were an infinite number of them. Liouville suggested that *e* was transcendental, and provided the first example of a provably transcendental number, now known as *Liouville's constant*.

$$\sum_{k=1}^{\infty} 1/10^{k!} = 0.110001000000000000000001000\ldots$$

where there is a 1 in place *n!* for *n* = 1, 2, 3, … and 0 everywhere else.

Problems involving partial differential equations of mathematical physics usually contain boundary conditions such as the condition that a vibrating string must be fixed at the endpoints. Liouville and his friend Charles Sturm, professor of mechanics at the Sorbonne, developed the so-called Sturm-Liouville theory. It is used in solving integral equations. In addition, Liouville also made contributions to differential geometry studying conformal maps (angle-preserving transformations). In all, Liouville wrote over 400 mathematical papers, 200 in number theory alone. He played a major role in bringing the work of Galois to general notice by publishing the latter's notes in 1846.

The roots of polynomial equations with rational coefficients are called algebraic numbers. All other real numbers are called transcendental numbers, because as Leonhard Euler put it, "they transcend the power of algebraic methods." In 1744 Euler became the first to recognize the distinction between algebraic and transcendental numbers. However no number was known to be transcendental, although it was suspected that π was. In 1873 Charles Hermite proved that *e* is transcendental and in 1882 Ferdinand Lindemann proved the same of π. As a result of Georg Cantor's work with transfinite numbers it has been shown that the transcendental numbers are not countable and thus are the most "numerous," even though at present we are more familiar with algebraic numbers.

When a game or sport is invented, their creators have no idea of the consequences of the rules they make for their game. These only become apparent later and lead to revisions in the rules to account for things that previously had not been thought about. The game of mathematics is similar. When human beings abstracted numbers from their surroundings they could have no idea of where it would all lead. As relations between numbers were noticed or imposed it became clear that humans were not totally in control of their inventions. Numbers seemed to have a life of their own with many dark corners of their personality that needed to be examined. At various times, mathematicians and others resisted this independent streak of numbers and made pronouncements worthy of holy bulls that certain unwelcome quantities did not exist as numbers. Declaring that they do not exist and enforcing the decree proved to be two completely different things. Fractions, negative numbers, irrational numbers, transcendental numbers, imaginary numbers, etc. all experienced this peculiar type of prejudice, but their intrinsic value eventually proved too valuable to ignore.

Quotation of the Day: "… one should strive above all else to avoid [obscurity] when treating the abstract and mysterious matters of pure Algebra. Clarity is, indeed, all the more necessary when one essays to lead the reader farther from the beaten track and into wilder territory." – Joseph Liouville