Adrien-Marie Legendre

French mathematician **Adrien-Marie Legendre** (September 18, 1752 – January 10, 1833), a disciple of Leonhard Euler and Joseph Lagrange, is remembered for the Legendre symbol and Legendre functions. He made substantial contributions to statistics, number theory, abstract algebra, and mathematical analysis, although most of his work was perfected by others, including Niels Abel, Jacques Hadamard and Charles de la Vallée Poussin. The former extended Legendre's work on elliptic functions and the latter two independently proved Legendre's conjecture of the prime number theorem, which can be stated as "the number of primes not exceeding x is asymptotic to $x/\log x$." Legendre was a member of the first committee to work on the definitions of the metric system of measurement. He published "Sur la méthode des moindres carrés," as an appendix to a book on determining the orbits of calculus. This was believed to be the first appearance of the method of least squares in print. It was a tremendous contribution to the development of statistics and was almost as revolutionary as calculus had been a century earlier.

The *least squares method* is a statistical means for finding a line or curve of "best" fit for a set of observed measurements. When the collected data are plotted as points on a graph and appear to fall near some line drawn through the midst of them, the distance of the points from the line varies with the line chosen. The average of the squares of these distances is taken as a measure of "goodness of fit" of the line. The "best-fit" line is that one for which this mean square deviation is minimal. It can be shown that the desired line must pass through the arithmetic mean $(\underline{x}, \underline{y})$ of the array. The least squares method can be extended to find a second-degree curve to fit a given set of data and generalized to other curves. Legendre developed his version in the context of geodesy for making arc measurements in determining the standard meter, taken to be 10^{-7} times the length of the terrestrial meridian quadrant through Paris at mean sea level.

Legendre was born in Toulouse, although his family moved to Paris soon after. There are few details of Legendre's early life and that would suit this retiring man. He once informed his colleague Siméon Poisson that he hoped if anyone spoke of him they would only speak of his work, which was his whole life. He was born into a well-to-do family and educated at the Mazarin College in Paris, where he studied mathematics and physics. On the advice of his teacher, the Abbé Joseph François Marie, and Jean Le Rond d'Alembert, he was appointed a professor at the École Militaire, where he taught with Laplace from 1775 to 1780, when he resigned. In 1782, he won the prize of the Berlin Academy for his essay on the path of a projectile, *Recherches sur la trajectoire des projectiles dans les milieux résistants*. The next year Legendre submitted his findings of his study of the attraction of ellipsoids. He introduced what are now known as Legendre functions, solutions to Legendre's differential equation, used to determine, via power series, the attraction of an ellipsoid at any exterior point.

The Legendre symbol is particularly useful in the number theory fields of factorization and quadratic residues. If p is a prime number and a is relatively prime to p, then the Legendre symbol (a/p) equals 1 if there exists an integer x such that $x^2 \equiv a \bmod p$, and -1 otherwise. The symbol enables an efficient formulation of the law of quadratic reciprocity. In 1783 Legendre was appointed an adjunct in the Académie des Sciences, filling the position vacated by Laplace. The brilliance of Legendre's work led to his appointment to the Anglo-French commission of 1787 to connect Greenwich and Paris geodetically. His three papers on geodesy were presented to the Académie in 1787 and 1788. In 1791 he was appointed a member of the committee of the Académie des Sciences, with the assignment of standardizing weights and measures, eventually bringing about the metric system. In 1795 he was appointed a professor at the École Normale.

Legendre's most famous book was the classic *Eléments de Géométrie* (1794), the leading elementary geometry textbook for 100 years, widely used as a substitute for Euclid's *Elements*. It was considered a pedagogical improvement because Legendre rearranged and simplified the propositions. The text, greatly admired on the continent, became the prototype of elementary geometry textbooks in the United States when John Farrar of Harvard University translated it into English in 1891. The celebrated man of letters, Thomas Carlyle, who as a young man taught mathematics, made another English translation of Legendre's geometry, which ran through 33 American editions. Later editions of Legendre's geometry contained elements of trigonometry as well as his proofs of the irrationality of π and π^2. In the 1803 edition Legendre included an appendix dealing with his attempts to prove the parallel postulate, a task with preoccupied him for 30 years. Even when Bolyai published his work creating non-Euclidean geometry, Legendre stubbornly clung to the belief that Euclidean geometry was the only certainty. He published his last article on the parallel postulate the year of his death.

In 1798 Legendre had published his *Essai sur la théorie des nombres* with appendices added in 1816 and 1825. It contained the proof of the law of quadratic reciprocity, due to Gauss, to whom Legendre gave proper credit. Gauss called the law of quadratic reciprocity "the gem of arithmetic." He gave eight different proofs of it in his works, the first when he was nineteen. It states that between the pair of congruences $x^2 \equiv q \pmod{p}$ and $x^2 \equiv p \pmod{q}$ in which p and q are distinct odd primes; either both congruences are solvable, or both are unsolvable, unless both p and q leave the remainder 3 when divided by 4, in which case one of the congruences is solvable and the other is not. Legendre consolidated all his number theory results in *Théorie des nombres* (1830). The results of his long study of elliptic functions were published in his *Exercises du Caclul Intégral* in three volumes in 1811, 1817, and 1819. During the period 1825 to 1832 he published the three-volume *Traité des fonctions elliptiques*, which completely reorganized his early work on elliptic functions.

Legendre was a timid individual who didn't put up much of a fight when a jealous Laplace condemned him to relative obscurity and denial of important positions or public recognition. Legendre had every right to be furious over the fact that Laplace, a far superior mathematician, had the audacity to appropriate some of Legendre's results without acknowledgement. Legendre took indignant exception when Gauss announced that he had made the discovery of the method of least squares and communicated it to other mathematicians several years before Legendre's 1806 publication. He fought for many years to have his priority in the matter recognized. In 1824 Legendre stood up to the government's attempt to dictate to the Institut National des Sciences et des Arts, which replaced the Académie des Sciences when it was closed in 1793. He refused to vote for the government's candidate for the Instut and as a result was deprived of his pension, forcing him to live his final years in poverty.

Quotation of the Day: "It is customary with Euclid, and various geometrical writers, to give the name *equal triangles*, to triangles which are equal only in surface; and to *equal solids*, to solids which are equal only in solidity. We have thought it more suitable to call such triangles or solids *equivalent*; reserving the determination *equal triangles*, or *solids*, for such as coincide when applied to each other."
– Adrien-Marie Legendre