# JOSEPH-LOUIS LAGRANGE

Both France and Italy claim **Joseph-Louis Lagrange** (January 25, 1736 – April 10, 1813), the greatest

analytical mathematician of his time, as their own. In 1788, he

published his masterpiece *Méchanique analitique* (Analytic

Mechanics), which is significant for its use of differential equations.

In it mechanics is developed algebraically and a wide variety of

problems are solved by the application of general equations. He laid

the foundation for finding a different way of expressing Isaac

Newton's Equations of Motion with what is now known as
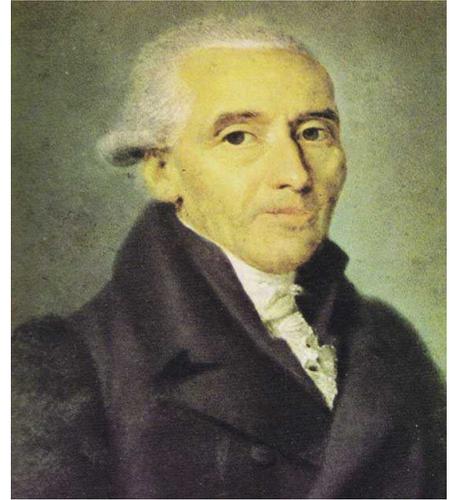
Lagrangian Mechanics. Given the subject we might find it odd that the book did not have a single

diagram or construction. But Lagrange explained: "One will not find figures in this work. The methods

that I expound require neither constructions, nor geometrical or mechanical arguments, but only

algebraic operations, subject to a regular and uniform course."


Lagrange was born in Turin, Sardinia-Piedmont (now Italy) and baptized Giuseppe Lodovico

Lagrangia. His father was once the King of Sardinia's war treasurer, but he lost his fortune in a series of

bad investments. Later Lagrange claimed that if he had been rich, he would have become an idler.

Lagrange's great-grandfather had been a French cavalry captain who left France to work in Turin for

the Duke of Savoy. Young Lagrange was educated at the college of Turin, but did not show any

particular talent for mathematics until he was about sixteen, at which time he read a memoir of Edmond

Halley, which inspired him and roused his enthusiasm for the analytic method. He went about the task

of studying mathematics on his own and was so successful that by the age of 19 he was so

accomplished a mathematician that he was appointed a lecturer at the Royal Artillery School in Turin.

Lagrange's mathematical reputation was established when, in 1755, he wrote a letter to Leonhard Euler in which he outlined his idea of a general method of dealing with "isoperimetric" problems, one of the motivating problems for the development of the calculus of variations, a name supplied by Euler in 1776. Euler, who had also researched the problem, recognized that the younger man's solution was superior to his own, and withheld publication of his result so Lagrange could get full credit for the invention. The Euler-Lagrange equation is a fundamental differential equation that gives the necessary conditions for solving classical problems dealing with paths, curves, and surfaces in the calculus of variations. Together with Euler, Jakob Bernoulli and Johann Bernoulli, Lagrange established the calculations of variations for dealing with mechanics. His applications of the calculus of variations to celestial mechanics were so monumental that by the age of 25 he was hailed by many as the greatest living mathematician.

When Euler left Berlin for St. Petersburg in 1766, he recommended that Lagrange succeed him as director of mathematics at the Berlin Academy of Sciences. Lagrange acceded to the wish of Frederick the Great, "the greatest king of Europe" to have "the greatest mathematician in Europe" resident at his court. Lagrange spent the next twenty years in Prussia where he produced on the average about one memoir a month, the most notable being the *Méchanique analitique*. With this work Lagrange transformed mechanics into a branch of mathematical analysis. Sir William Hamilton described the *Méchanique analitique*, which is the basis for all later work in the field, as a "scientific poem."

After Frederick's death, Lagrange left Berlin for Paris to become a member of the Académie des Sciences in Paris. In 1790, Lagrange was appointed to the committee of the Académie charged with standardizing weights and measures. The committee worked on the metric system with Lagrange advocating a decimal basis. In 1793, the Reign of Terror banned all learned societies. The weights and measures commission, with Lagrange as its chairman, was the only one allowed to continue.

In 1793, a law was passed ordering the arrest of all foreigners born in enemy countries and all their property confiscated. Antoine Lavoisier, the father of modern chemistry, intervened on behalf of Lagrange and the latter was granted an exception. Tragically, Lavoisier was not able to save himself. On May 8, 1794, at a trial lasting less than a day, Lavoisier and 27 other scientists were condemned to death, and were guillotined the afternoon of the trial. Lagrange said of Lavoisier: "It took only a moment to cause his head to fall and a hundred years will not suffice to produce its like." When Napoleon Bonaparte took power, he greatly encouraged scientific studies in France and became a liberal supporter of Lagrange, whom he called "the lofty pyramid of the mathematical sciences." Napoleon named Lagrange to the Legion of Honor and made him Count of the Empire in 1808.

Lagrange made contributions to all fields of mathematics, including differential equations, Taylor series, the calculus of variations, probability, and theory of equations. He was the first to develop today's familiar methods of finding *maximum* and *minimum* values of functions. He was only nineteen when he devised these methods, and he regarded them as his best mathematical work. One of his favorite computational and theoretic tools was "integration by parts," familiar to calculus students. He put his mathematical discoveries and inventions to use in solving the great problems of his day in mechanics and astronomy. In physics, he was concerned with wave theory, the propagation of sound, and the problem of hydrodynamics. In astronomy, he worked on the problems of the moons of Jupiter, the shape of the earth, the stability of the solar system, and the orbits of comets.

Despite a lifetime of interesting and fruitful work, Lagrange spent a lonely and melancholy life after the early death of his first wife. At the age of fifty-six, the daughter of the astronomer Lemonnier, nearly forty years younger than he, insisted on marrying him. She proved a devoted companion, one whom he could barely stand being away from for even the shortest period. No longer lonely, he found

new bursts of energy for his mathematical and scientific explorations. This continued even in his 70s, but then his health began to fail and he experienced fainting spells. In 1813, Lagrange was named Grand Croix of the Imperial Order and died a week later. Two days before his death, Gaspard Monge and other friends wishing to bid him farewell visited him. Shortly after they left, he experienced a fainting spell from which he did not recover, dying on the morning of April 10, 1813. Lagrange is buried in the Panthéon in Paris.

As something of a footnote to the life and accomplishments of Lagrange, the following account of the adoption of the metric system is worth telling. Not only did the Revolutionary French government accept the recommendations of Lagrange and the Royal Academy, they made the use of the metric system of weights and measures mandatory in 1795. But they went further. They also decreed a new series of units of time and a new way of recording time. By the Act of November 24, 1793, a radically new calendar and an almost equally new system of measuring time were proclaimed. The following table gives the system of units used. Notice the desperate attempt to decimalize time.

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| 100 seconds | = 1 minute |
| 100 minutes | = 1 hour |
| 10 hours | = 1 day |
| 10 days | = 1 week or decad |
| 3 weeks = 30 days | = 1 month |
| 12 months plus 5 or 6 carnival days | = 1 year |

Since the First French Republic began on November 22, this date was regarded as the commencement of the new era. The Julian calendar and all its terminology were abolished. The months and the days of the week were given new names. The French people, for all their revolutionary zeal, could not endure

such a violation of their customs and on April 7, 1795 (or 8 Germinal, 3 by the revolutionary calendar) the division of the day into 100,000 parts was suspended. Finally, when Napoleon became emperor, it was decreed that 11 Nevose, 14 should become January 1, 1806 (as it always was as far as the people were concerned).

**Quotation of the Day:** "As long as algebra and geometry proceeded along separate paths, their advance was slow and their applications limited. But when the sciences joined company, they drew from each other fresh vitality and thenceforth marched on at a rapid pace." – Joseph-Louis Lagrange