ERNST EDUARD KUMMER

Prussian born **Ernst Eduard Kummer** (January 29, 1810 – May 14, 1893) was one of the mathematicians who significantly contributed to the eventual resolution of Fermat's Last theorem. He was the son of a physician who died when Kummer was only three. Kummer entered the University of Halle in 1828 to study theology. Fortunately, part of his training was in mathematics, so that he might better understand philosophy. Kummer soon chose mathematics as his main course of study. In 1831 he was awarded a doctorate as a result of having won a prize for a mathematical essay he wrote on a topic given him by his mathematics lecturer H.F. Scherk.

In 1832, Kummer moved to Liegnitz, now in Poland, teaching there for ten years in the Gymnasium, where his most famous student was Leopold Kronecker. At this time the simple schoolteacher made significant advances in mathematical research. Eminent mathematicians, including Karl Jacobi and P.G. Lejeune Dirichlet, with whom Kummer shared his mathematical discoveries, arranged for him to be appointed Professor of Mathematics at the University of Breslau in 1842, where he taught until 1855. In that year, Kummer succeeded Dirichlet at the University of Berlin. There, with Kronecker and Karl Weierstrass, he helped establish Berlin as one of the leading mathematical centers in the world. A capable applied mathematician, Kummer also trained German army officers in ballistics at the Berlin War College.

Kummer's most important mathematical achievement was the introduction of "ideal" numbers. His motivation was the desire to solve Fermat's Last Theorem (i.e. the assertion that no whole numbers or fractions x, y, z exist such that

$$x^n + y^n = z^n$$

if n is a whole number greater than 2). He created the complex number system and was able to prove that Fermat's Last Theorem was true for an infinite number of exponents, those that are divisible by "regular" primes. It will serve no useful purpose to elaborate on the distinction between "regular" primes and "irregular" primes, which E.T. Bell, in his article on Kummer in *Men of Mathematics,* called certain slippery "exceptional primes." The only primes less than 100 that are not regular are 37, 59, and 67.

As a result of Kummer's work, Fermat's Last Theorem was shown to be true for all exponents n less than 100. His attempt to prove the theorem in general broke down because the unique factorization of integers (every positive integer $z \geq 2$ can be expressed uniquely, without respect to order, as the product of powers of prime numbers) does not extend to other rings of complex numbers. Kummer attempted to restore the uniqueness of factorization by introducing "ideal" numbers. Regrettably, it is not possible to give a simple non-technical explanation of Kummer's "ideal" numbers. There are two versions of the story of Kummer being awarded a monetary prize in a competition he did not enter. One claims that the French Academy of Sciences had been holding the prize in reserve hoping to award it to the one who completely solved Fermat's Last Theorem, but apparently decided that Kummer's accomplishment was the next best thing. The other version is that since no one had submitted a paper worthy of the Academy's "Grand Prize" during the period 1853 – 1856, it was decided, in 1857, to award the prize to Kummer for his work with the complex roots of unity.

Speaking of prizes for solving Fermat's Last Theorem brings to mind the name of Paul Friedrich Wolfskehl (1856 – 1906), the son of a wealthy banker who after taking a medical degree developed multiple sclerosis. Realizing that his handicap would prevent him from practicing medicine he turned to the study of mathematics. In 1883, at Berlin, he attended lectures of the then 77-year-old Kummer,

who excited his interest in number theory, particularly algebraic number theory. Wolfskehl became aware of Fermat's Last Theorem by studying Kummer's relevant papers. Whether he made an attempt to solve the theorem is not known, but it seems likely he gave it a try. At any rate when he died, his will provided a prize of 100,000 marks to be given to the first person to prove the theorem, who turned out to be Andrew Wiles.

In his 1969 book *3.1416 and All That*, which he wrote with William G. Chinn, Philip J. Davis related an interesting story told to him about Wolfskehl by Alexander Ostrowski. It seems that despondent over his inability to solve Fermat's Last Theorem and a failed romance, Wolfskehl decided to commit suicide. He carefully went about putting his affairs in order, making his will and writing final letters to friends. He had chosen a precise time for doing the deed and found he had an hour or so to kill before the appointed time. He went into his library and idly picked up the latest article written by Kummer. As he pored over it, he felt he had found an error in Kummer's argument and set about checking the doubtful point. Several hours passed before he concluded that Kummer had been correct, but as the time to take his life had also passed, he decided to live and return to mathematics.

Despite being called the father of modern arithmetic (that is, number theory), Kummer was rather poor at simple arithmetic. Once, in a class, he needed to find the product of seven and nine. "Seven times nine," he began, "Seven times nine is er – ah --- ah --- seven times nine is …." "Sixty-one," a student suggested. Kummer wrote 61 on the board. "Sir," said another student, "it should be sixty-nine." "Come, come, gentlemen, it can't be both," Kummer exclaimed. "It must be one or the other." Paul Erdős had another version of this story. Kummer calculated 7 x 9 as follows: "Hmmm the product cannot be 61, because 61 is prime, it cannot be 65, because 65 is a multiple of 5, 67 is a prime, 69 is too big. Only 63 is left."

Quotation of the Day: “A peculiar beauty reigns in the realm of mathematics, a beauty which resembles not so much the beauty of art as the beauty of nature which affects the reflective mind, which has acquired an appreciation of it, very much like the latter.” – Ernst Eduard Kummer