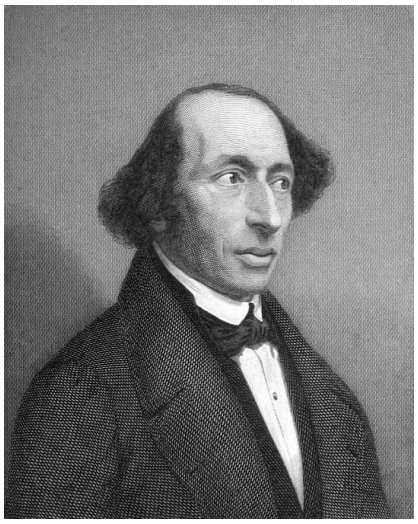
CARL GUSTAV JACOB JACOBI

Along with Norwegian Niels Abel, German mathematician **Carl Gustav Jacob Jacobi** (December 10, 1804 – February 18, 1851) was one of the founders of the theory of elliptic functions, which he described in his 1829 work *Fundamenta Nova Theoriae Functionum Ellipticarum* (New Foundations of the Theory of Elliptic Functions). His achievements in this area drew praise from Joseph-Louis Lagrange, who had spent some 40 years studying elliptic integrals. Jacobi also investigated number theory, mathematical analysis, geometry and differential equations. His work with determinants, in particular the Hamilton-Jacobi theory, a technique of solving a system of partial differential equations by transforming coordinates, is important in the presentation of dynamics and quantum mechanics. V.I. Arnold wrote of the Hamilton-Jacobi method, "… this is the most powerful method known for exact integration."



Born in Potsdam, Jacobi was the son of a prosperous Jewish banker. His older brother Moritz Hermann Jacobi was a well-known physicist and engineer. The latter worked as a leading researcher at the Academy of Sciences in St. Petersburg. During their lifetimes Moritz was the better known of the two for his work on the practical applications of electricity and especially for his discovery in 1838 of galvanoplastics. Also called electrotyping, it is a process something like electroplating for making duplicate plates of relief, or letterpress, printing. Carl was constantly mistaken for his brother or even worse congratulated for having such a distinguished and accomplished brother. To one such compliment he responded with annoyance, "I am not his brother, he is mine."

Carl Jacobi demonstrated great talent for both languages and mathematics from an early age. He received his initial education at home from one of his maternal uncles before entering the Gymnasium just prior to his twelfth birthday. After only one year there he met all the requirements of admission to the university except age. He had to wait until he was 16 to enter the University of Berlin. In the interim he read advanced texts of the leading mathematicians of the day. Since at the time, German universities were not recognized for mathematical excellence, Jacobi was pretty much left to his own devices in studying mathematics while there. By the end of 1824, Jacobi passed the examinations necessary to teach mathematics, Latin and Greek in the schools. Despite his Jewish faith he was appointed to a position at the Joachimsthalsche Gymnasium, one of the top schools in Berlin. From the very beginning it was evident that Jacobi was a born teacher. Later, as he developed his own mathematical ideas with amazing rapidity, he was hailed as the most inspiring teacher of his era. Jacobi adopted a seminar style of teaching in which he presented his own discoveries, allowing his students to participate in the creation of new ideas. Jacobi believed it was a waste of talent for good students to work endless hours so that they might master the work of others.

In 1825, Jacobi obtained his doctorate for a dissertation on continued fractions. That same year, he presented a paper on iterated functions to the Academy of Sciences for his habilitation thesis. Around this time Jacobi converted to Christianity in order to be eligible to obtain a university post. In 1826 he became extraordinary professor of Mathematics at Königsberg, and two years later he was promoted to ordinary professor. He occupied this chair until 1842 when he resigned due to poor health after several collapses brought on by overwork. He was awarded a generous pension from the Prussian Government, and moved to the University of Berlin, where he gave occasional lectures, remaining until his death.

Jacobi's most celebrated investigations were those on elliptic functions, and their modern notation is

substantially due to him. He established the theory of these functions simultaneously but independently with Niels Abel. Jacobi's results were given in his *Functionum Ellipticarum* and later papers published in *Crelle's Journal*. Like Abel, Jacobi recognized that elliptic functions were not merely a collection of integration theorems, but were new types of functions. Jacobi applied elliptic functions to number theory, using them to prove Pierre de Fermat's polygonal number theorem: "any integer is the sum of the squares of at most four integers." With his work on celestial mechanics, Jacobi introduced what is known as the Jacobi integral for a sidereal coordinate system. He was one of the early founders of the theory of determinants and did much to bring determinants into general use. In a long memoir *De determinantibus functionalibus* (1841), he introduced the Jacobian. It is an n x n determinant formed from a set of n functions in n unknowns. Although not the first to use the term Jacobian [it appeared in a paper of Cauchy in 1815], Jacobi proved that the Jacobian is zero if and only if the functions are related. In *Men of Mathematics* by E.T. Bell labeled Jacobi "The Great Algorist," which, if we consider his accomplishments, seems an appropriate title.

In 1848, with revolutions sweeping Europe, Jacobi unwisely allowed himself to be persuaded to run for parliament. He not only was defeated, the decision annoyed his royal patron, the King of Prussia, who promptly cut off Jacobi's pension. Although Jacobi inherited a small fortune from his wealthy father, a severe Europe-wide depression wiped out all his funds. A friend took care of Jacobi's wife and seven children while he moved into a dingy hotel to continue his mathematical researches. He was diagnosed with diabetes and was advised to spend time in Italy where the climate would help him recover. Unable to afford to do so, Jacobi was saved when Dirichlet, strongly supported by Alexander von Humboldt, arranged for a grant from Kaiser Friedrich Wilhelm IV to pay his expenses. The climate of Italy revitalized his health, allowing him to return to his research, but not sufficiently to teach. In January 1851, Jacobi contracted influenza and, then before he had regained his strength from the first illness,

smallpox. He died a few days later.

For some forty years Adrien-Marie Legendre labored on calculating elliptic integrals of the form $\int R(x, \sqrt{S})\, dx$ where S is either a third or fourth degree polynomial and $R(x, \sqrt{S})$ is a rational function of x and $\sqrt{S}$. These integrals were named *elliptic* after Lagrange and Euler found that they could be used to find the arc length of an ellipse. Abel and Jacobi found that working with inverse functions of integral equations made the whole subject considerably simpler. These inverse functions are called *elliptic functions*. One of the important properties of elliptic functions is their *double periodicity* (discovered by Abel in 1825); that is, if $E(x)$ is an elliptic function, then there are two distinct numbers, say p and q, such that $E(x - p) = E(x)$ and $E(x + q) = E(x)$ for all values of the variable x.

The work of Abel and Jacobi with elliptic functions launched the growth of the larger field of the theory of functions of a complex variable. Private papers of Carl Friedrich Gauss showed that he anticipated the discoveries of Jacobi and Abel by as much as 27 years. Jacobi was so impressed with the important results gained by a simple inversion of the functional relationship in elliptic integrals, he gave his advice for mathematical success: “you must always invert.”

Quotation of the Day: “Mathematics is the science of what is clear by itself.” – Carl Jacobi