William Rowan Hamilton

William Rowan Hamilton (August 4, 1805 – September 2, 1865) was one of the most original and creative mathematicians of his time. He made influential contributions to optics and in the foundations of algebra. His earliest work on optics was published in *The Theory of Systems of Rays* (1828). He became internationally celebrated in 1832 when he theoretically predicted the existence of conical refraction, asserting that light would be refracted in the form of a hollow cone through a specially cut crystal of Iceland spar. When Humphrey Lloyd, Professor of Physics at Trinity College, Dublin, confirmed this experimentally two months later, it caused a sensation in the scientific community. Hamilton's achievements led to his being knighted in 1835. He is best known for developing the mathematical theory of quaternions, numbers that are anticommutative, that is numbers a and b such that $a \cdot b = - b \cdot a$. Anticommutative algebras and his formulation of mechanics were later incorporated into the equations of quantum mechanics.

Hamilton was born and died in Dublin. His father was a solicitor, often away in England on legal business. When Hamilton was three he was sent to Trim to be raised by his aunt and his uncle, Reverend James Hamilton. The latter was a bit of an eccentric with his own peculiar ideas about education. He made a hole in the wall of his bedroom and ran a string through it, the end of which he tied to his nephew's toe. Early in the morning, he only had to tug on the string to alert young Hamilton that it was time to begin his studies. A prodigy, young Hamilton learned Latin, Greek and Hebrew by the time he was five and at 13 he had mastered as many languages as years he had lived, including Italian, French, Sanskrit, Persian, Arabic and Indian dialects.

At age 10, Hamilton came across a Latin copy of Euclid's *Elements*, which led him to a serious study of geometry. The young genius was introduced to modern analysis by reading a copy of *Arithmetica Universalis*, which were based on Newton's lectures deposited in the University Library. His successor as Lucasian Professor at Cambridge, William Whiston published an unauthorized edition in 1707. Newton so disliked it that he refused to have his name appear as author (he even considered buying the whole edition so that he could destroy it). An English translation of the work by Joseph Raphson appeared as *Universal arithmetick* in 1720. Hamilton next read Newton's *Principia* and the four-volumes of Laplace's *Mécanique céleste*. When Hamilton discovered a mistake in the latter work, he wrote a paper on the matter that attracted considerable attention. John Brinkley, the Astronomer Royal of Ireland, remarked: "This young man, I do not say will be, but is the first mathematician of his age." It was widely held that a new Newton had arrived on the scene.

Hamilton entered Trinity College, Dublin at age 18 and became one of the very few individuals to obtain the highest grade in two subjects: Greek and mathematical physics. While still an undergraduate, the chair of Astronomy at Dublin became vacant and Hamilton was appointed to fill it. He was also made Astronomer Royal of Ireland so he would be free to pursue his own research. He lived and worked at the Dunsink Observatory. Hamilton unified the field of optics under the principle of varying action, and adapted the methods he developed for the study of optical systems to the study of dynamics, which has become of fundamental importance in modern physics, in two celebrated papers on *General Methods in Dynamics* (1834, 1835).

In the work that Hamilton considered his grandest achievement, he considered the complex number $a + bi$ as an ordered pair (a,b), and showed that rotations in a plane are described by the algebra of such couples. He showed that the sum of two complex numbers could be represented by a parallelogram, and that complex numbers could be a useful tool in plane geometry. Hamilton coined the word "vector," when he showed it was possible to deal with lines in all possible positions and directions, free from dependence on Cartesian axes of reference. Regarded as one of the great "abstractors" of algebra, he also introduced the del (∇), a vector differential operator. Hoping to develop a three-dimensional analogue of complex numbers he conceived the idea of *quaternions*, which became an obsession that plagued him for the rest of his life. These were ordered triples with which he planned to describe rotations in three dimensions. In developing the algebra of quaternions he had no problem in adding and subtracting triples, but how to multiply them to allow division continued to escape him.

Every morning his children asked, "Well, Papa can you multiply triples?" and he had to confess he could not. One October day in 1843 the idea of using four dimensions came to him in a flash as he walked by the Royal Canal of Dublin. He took out his penknife and scratched the fundamental formula $i^2 = j^2 = k^2 = ijk = -1$ into the stone of Brougham Bridge [Figure 8.2]. In 1958, a plate was erected on the site, commemorating the discovery and displaying the fundamental formula [Figure 8.3].

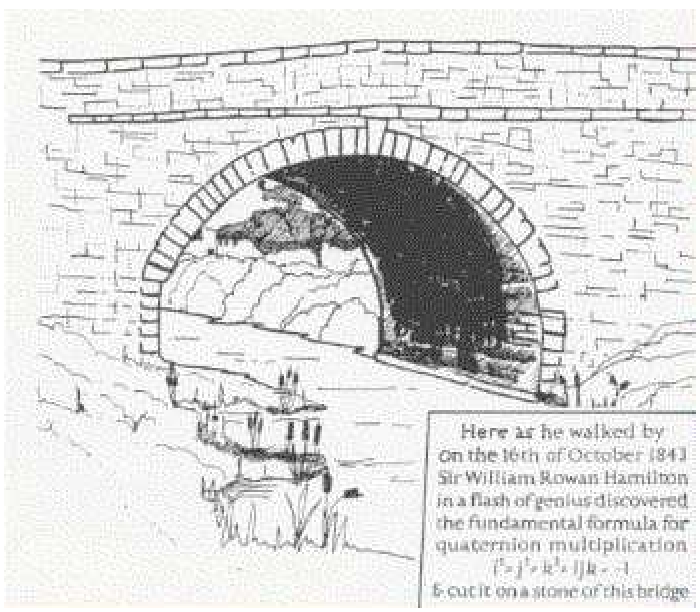

Figure 8.2

Figure 8.3

Hamilton was certain that his discovery would revolutionize mathematical physics and spent the rest of his life trying to convince the world of the value of his invention. Quaternions were superseded in most applications by the methods of vector and tensor analysis. Although his quaternions did not have the expected value in mechanics and geometry, his discovery that in their algebra, multiplication is not commutative, led later mathematicians to discover many new types of abstract algebras. In his article "If Hamilton Had Prevailed: Quaternions in Physics," *The Mathematical Intelligencer,* 17, 4, 1995, Joachim Lambek demonstrated how certain key results in theoretical physics could be expressed concisely in the language of quaternions.

Another mathematical area in which Hamilton made significant contributions was graph theory. A *Hamiltonian path* is one that visits each vertex exactly once. If it returns to the initial vertex of the path it is called a *Hamiltonian cycle*. A *Hamiltonian graph* is a graph that contains a Hamiltonian cycle. Hamilton developed a game that he sold to a toy manufacturer. It consists of a two-dimensional wooden representation of an icosahedron with the 20 corner points (vertices) labeled with the names of important cities. The objective of what Hamilton called the *Icosian game*, was to find a cycle along the edges of the solid so that each city was on the cycle exactly once. Figure 8.4 is a photograph of one of the only three known examples of Hamilton's puzzle.

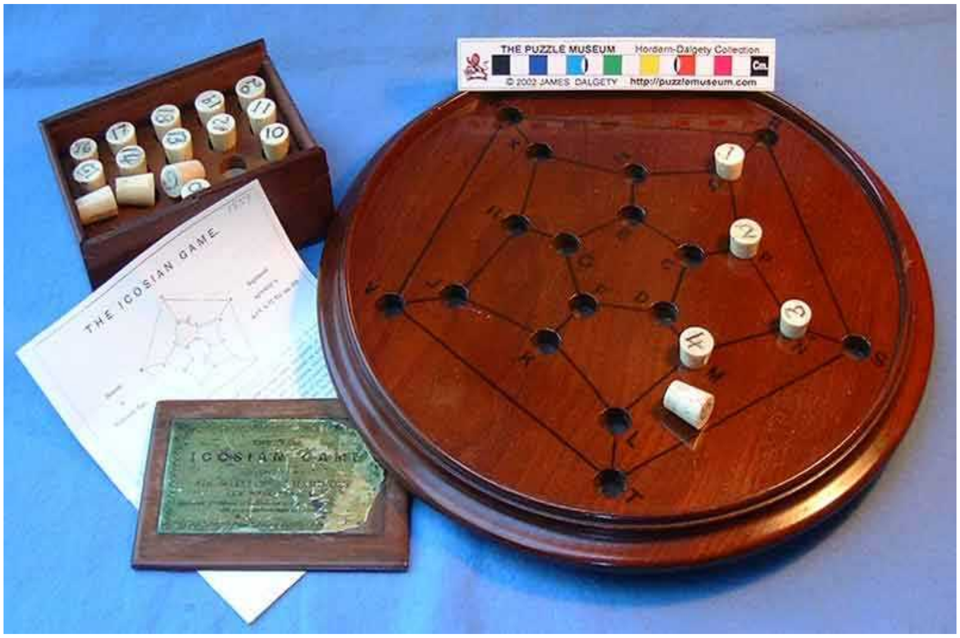

Figure 8.4

In the latter part of his life Hamilton became a recluse, continuing to work and drinking excessively. He made a spectacle of himself at a meeting of the Geological Society due to his intoxication. Things did not improve in the following years, but he was able to complete his *Lectures on Quaternions* (1853). Hamilton's exposition made it very difficult for readers to follow his arguments and understand his results. He recognized the book's shortcomings and slaved to produce a revision, *Elements of Quaternions.* At 400 pages it was double the length he planned and took five more years than he expected. The final chapter was incomplete when he died. His death, the result of a severe attack of gout, occurred shortly after learning that he had been elected the first foreign member of the National Academy of Sciences in the United States. Hamilton's son, William Edwin, published the work with a preface of his own. It wasn't only his mathematical work that caused Hamilton to feel unappreciated.

He also fancied himself quite a poet. He frequently sent verses to his friends William Wordsworth and Samuel Coleridge. Wordsworth is given credit for trying to discourage Hamilton from writing bad poetry and to spend his considerable talents in more productive pursuits.

Friends described Hamilton as having “a buoyant cheerfulness and a kindly human-heartedness.” Women found him charming and attractive because of his great intelligence. However, he wasn’t much of a success in the romantic area. In August 1824 Hamilton visited the Disney family and fell in love with their daughter Catherine. Since he had three more years at Trinity, he was not in a position to propose marriage. The following February Catherine married a clergyman, fifteen years senior because her family felt he could offer her a comfortable life. After Hamilton was appointed to the chair of Astronomy, Catherine came to visit him at the Observatory. He was so nervous in her presence that he broke the eyepiece of the telescope as he demonstrated its use to her.

Since Hamilton believed he should be married, but could not marry his true love, he reasoned it didn’t really matter whom he married. He settled on Helen Maria Bayly who conveniently lived at a farm near the observatory. It was not a love match and as Helen wasn’t much of a housekeeper and was often ill, she added little to his life, save three children.

In 1848 Catherine wrote to Hamilton, beginning a correspondence that lasted six weeks. They shared such personal sentiments that Catherine, feeling guilty, confessed to her husband. Hamilton pledged to the clergyman that he would write no more, which led the distraught Catherine to unsuccessfully try to commit suicide. After her recovery, Catherine spent the rest of her life apart from her husband caring for her mother and siblings. Hamilton did not keep his promise and continued corresponding with Catherine through her sister. In 1853 he received a pencil case from Catherine inscribed, “From one whom you must never forget, nor think unkindly of, and who would have died more contented if we had once more met." Hamilton hurried to Catherine and gave her a copy of *Lectures on Quaternions*, which might not be considered a very romantic present for the critically ill love of one's life, but since he believed he was sharing with her his greatest achievement, maybe it was. Catherine died two weeks later. Hamilton was inconsolable and in his grief wrote as many as two letters a day to members of Catherine's family.

Quotation of the Day: "Time is said to have only one dimension, and space to have three dimensions. … The mathematical quaternion partakes of both of these elements; in technical language it may be said to be "time plus space", or "space plus time," and in this sense it has, or at least involves a reference to, four dimensions." – Sir William Rowan Hamilton