Marie-Sophie Germain

Marie-Sophie Germain (April 1, 1776 – June 27, 1831), called Sophie, was born in Paris at the time of revolution, but the French Revolution didn't help talented, curious women wishing to enter a domain considered the exclusive province of men. She was the daughter of a prosperous silk-merchant, who constantly tried to discourage her desire to study mathematics. Despite revolution and the subsequent Reign of Terror, her bourgeois family's prosperity was not greatly affected. As Germain never married or had any means of earning a living, she remained at home her entire life.

To be safe from the dangers in the streets, Sophie lurked about her father's library. At age thirteen, she read an account of the death of Archimedes, who was killed when, so engrossed in his diagrams, he failed to respond to the orders of a Roman soldier. Inspired that someone could be so captivated by a problem, she decided that mathematics must be the most fascinating subject of all. Her first biographer, Guillaumme Libri, speculated that she might have seen in the study of mathematics "an environment where she could live untouched by the confusion of social reality." Whatever her reasons, Germain went about her plan with a vengeance. She taught herself Latin and Greek so she might read ancient mathematical works. Believing that mathematics was an inappropriate interest for females and wishing to discourage her from reading the works of Newton and Euler in her bedroom at night, her father took away her fire, light and clothes. Undaunted, she bundled herself in blankets and read by the light of filched candles while her parents slept.

Eventually Germain's father lessened his opposition to her study of mathematics. When she was eighteen, the École Polytechnique was established as part of the revolution's educational reforms, but these didn't extend to admitting female students. Although Germain was denied the privilege of attending the École, she was able to obtain the lecture notes of some male students. Assuming the identity of a male student, "M. Antoine August LeBlanc," Germain submitted a memoir on analysis to Joseph-Louis Lagrange, which he found so outstanding he wanted to meet its author. When he discovered the paper was the original work of a woman, he became her sponsor and introduced her to the company of mathematicians and scientists that she could never have aspired to meet on her own. Since she was could not take the route of male students, she proceeded on her own course of study with the result that her mathematics education was somewhat disorganized and directionless.

Intrigued with Carl Friedrich Gauss' number theoretic work in his *Disquisitiones Arithmeticae*, Germain wrote a dozen letters to him between 1804 and 1809, describing her own efforts in the field, including her attempts to solve Fermat's Last Theorem (FLT). She again used the pen name "M. LeBlanc," fearful that if it were known she was a woman, she would be ignored. Gauss praised her proofs and spoke highly of M. LeBlanc's accomplishments in letters to other mathematicians. The two never met, and it wasn't until after the French troops occupied Gauss' hometown of Braunschweig that he learned her true identity. Concerned that Gauss might experience Archimedes' fate, Germain contacted a family friend, General Pernety, gaining his assurance that no harm would befall the great mathematician. When Gauss was told that the intervention on his behalf was due to Sophie Germain, he insisted he knew no such person. Finally she wrote, confessing that she was "M. LeBlanc." When Gauss learned that his correspondent was a woman he was even more generous in his praise.

> "A taste for the abstract sciences in general and above all the mysteries of numbers is excessively rare: one is not astonished at it; the enchanting charms of this sublime science reveal themselves only to those who have the courage to go deeply into it. But when a person of the sex, which,

according to our customs and prejudices, must encounter infinitely more difficulties than men to familiarize herself with these thorny researches, succeeds nevertheless in surmounting these obstacles and penetrating the most obscure parts of them, then without doubt she must have the noblest courage, quite extraordinary talents and a superior genius. Indeed nothing could prove to me in so flattering and less equivocal manner that the attractions of this science, which has enriched my life with so many joys, are not chimerical, as the predilection with which you have honored it."

Germain wrote to Adrien-Marie Legendre concerning her work on Fermat's Last Theorem. Impressed, he credited her with what is now known as *Germain's Theorem* in a footnote in the second edition of his book *Essai sur la Théorie des Nombres*. The theorem states that in the case when n is equal to certain prime numbers, now called *Germain primes*, the equation $x^n + y^n = z^n$ probably has no solutions. Germain primes satisfy the condition that if n is prime, then $2n + 1$ must also be a prime. The first few Germain primes are 2, 3, 5, 11, 23, 29, 41, 53, 83, 89, and 113. In an elegant proof, she showed that if n as such a prime, then one of the three values x, y or z must be a multiple of n, greatly restricting the possible values for a solution. Not having access to computers, Germain had considerable difficulty in determining when p and $2p + 1$ are both primes. As of 2003 the largest known Germain prime consists of 34,547 digits. Germain's theorem separated FLT into two cases, for numbers not divisible by 5 and for those that were divisible by 5. As a result the possible instances when FLT might fail for primes less than 100 were reduced to case number two. In 1825, Legendre and P. G. Lejeune Dirichlet independently based their proofs that the case of $n = 5$ had no solutions using her techniques. In 1839 Gabriel Lamé made some ingenious additions to Germain's method and proved that FLT had no solutions in the case of $n = 7$.

When German physicist Ernst F.F. Chladni visited Paris in 1808, he demonstrated the curious patterns produced on small glass plates covered with sand, which could be played like violins with a bow. The

sand moved about until it reached the nodes. Napoleon authorized the French Academy of Sciences to offer a prize for the best mathematical explanation of this phenomenon. The challenge was to "formulate a mathematical theory of elastic surfaces and indicate how it agrees with empirical evidence." Most mathematicians ignored the contest, believing the mathematics needed to formulate such a theory didn't exist. Germain's anonymous 1811 entry was the only one, and her lack of a formal education was evident, so she did not win the prize. The judges found mathematical flaws in her argument, noting that she had not derived her hypothesis from principles of physics. Germain's self-study didn't include the mathematical methodologies appropriate to the molecular structure theory for materials, then in vogue. However, these methods were of no use in explaining the phenomena in question, so Germain brought original insights to the problem.

The judges, especially Lagrange, admired Germain's originality. He corrected an error in her calculations and devised an equation that he thought might describe Chladni's patterns. The deadline for submitting theories was extended two more years. Once again Germain was the only one to submit an entry. In it, she demonstrated that Lagrange's equation yielded Chladni's patterns in several cases, but because she was not able to offer a satisfactory derivation of the equation from physical principles, her work received only an honorable mention. The contest was reopened in 1815, at which time Germain's third attempt was deemed worthy of the prize, although still considered lacking in rigor. Though this was the high point of her scientific career, she did not attend the award's ceremony, believing that the judges did not fully appreciate her work and that the scientific community did not show her the proper respect. Germain continued to work in mathematics until her death from breast cancer, at age 55. She died before an honorary doctorate in mathematics from Göttingen, arranged for her by Gauss, could be awarded.

Quotation of the Day: "Algebra is but written geometry and geometry is but figured algebra." –

Sophie Germain.