Carl Friedrich Gauss

German mathematician, astronomer and physicist [**Johann] Carl Friedrich Gauss** (April 30, 1777 - February 23, 1855) was one of the greatest mathematicians who ever lived. In *Men of Mathematics,* E.T. Bell called him the "prince of mathematicians." To this day his work still dominates the mathematical world. He published more than 150 works, making important contributions to the "least squares method," Gauss-Jordan elimination for solving matrix equations, and the bell curve for normal distribution. He laid the foundation for number theory, and in 1801 rediscovered the lost asteroid Ceres using advanced computational techniques. Gauss applied rigorous mathematical analysis to such subjects as geometry, geodesy, electrostatics, and electromagnetism. He was involved in the first worldwide survey of the Earth's magnetic field. Between 1796 and 1800, mathematical ideas came so often and so quickly to him that he barely had time to write them all down. He was not one to rush to publish. In fact he did not consider that hasty spreading of the news of breakthroughs in mathematics was as necessary as it might be in the various sciences. He saw no excuse for publication of "slovenly or ill-digested work." One of his mottos says it best. His intention was to publish "Few, but ripe." After his death, many important papers were found that he had not published because they had not met his high standards.

Gauss was born in the Duchy of Brunswick, the son of a domineering, illiterate, unkind and uncouth bricklayer. His mother was his father's second wife, and their marriage was not happy. She lived to the age of 97, always devoted to her son. Gauss was an astonishing prodigy who taught himself to read and to calculate before the age of three. Had not his elementary school teacher, Herr Buttner and his assistant Johann Martin Bartels [who later taught Nikolai Lobachevsky] taken note of Gauss's precocity, he might have become a laborer rather than one of the foremost mathematicians of all time. His teachers convinced the Duke of Brunswick-Wolfenbuttel to provide the 14-year-old genius with a stipend to pay for his education. In 1792, Gauss was sent to the Brunswick Collegium Carolinum, where he perfected his skill with ancient and modern languages and familiarized himself with elementary geometry, algebra, and analysis. By this time he had established his life's work habits: extensive empirical investigation, which led to conjectures and new insights that guided further experiments and observations.

Gauss formulated the principle of least squares, while adjusting unequal approximations and searching for regularity in the distribution of prime numbers, and found results that would hold if "Euclidean geometry were not the true one." He used induction to prove the law of quadratic reciprocity, unaware, due to his limited access to mathematical works, that Legendre had already published it in 1795. Gauss was so taken with the theorem that he gave seven different proofs of it over the years. In *Göttingischegelehrie Anzeigen*, March 1817, he asserted the desirability of not merely finding a proof of theorems, but finding more elegant proofs:

> "It is characteristic of higher arithmetic that many of its most beautiful theorems can be discovered by induction with the greatest ease but have proofs that lie anywhere but near at hand and are often found only after many fruitless investigations with the aid of deep analysis and lucky combinations…it often happens that many theorems, whose proof for years was sought in vain, are later proved in many different ways. As soon as a new result is discovered by induction, one must consider as the first requirement the finding of a proof by any possible means… sometimes one does not at first come upon the most beautiful and simplest proof, and then it is just the insight into the wonderful concatenation of truth in higher arithmetic that is the chief attraction for study and often leads to the discovery of new truths. For these reasons the finding of new proofs for known truths is often at least as important as the discovery itself."

Gauss went to Göttingen, where he devoured mathematical classics unavailable to him at the Collegium. His only friend was Farkas Bolyai, with whom Gauss corresponded for many years. Originally he planned to become a philologist, but in 1796 he made a dramatic discovery that tipped the balance towards life as a mathematician. He obtained the necessary and sufficient conditions for the constructability with compass and straightedge of a regular 17-gon, the first advance in Euclidean constructions in two millennia. Gauss left Göttingen without a diploma and turned to tutoring to earn his living. The Duke offered to continue the stipend if Gauss would submit a doctoral dissertation to the University of Helmstedt. His thesis, supervised by Johann Friedrich Pfaff, had the lengthy title *A New Proof of the Theorem that Every Integral Rational Algebraic Function can be Decomposed into Real Factors of the First or Second Degree*; that is, he offered a proof of the fundamental theorem of algebra. Because of the Duke's stipend, Gauss didn't need to find a position, so he devoted his time to research.

His famous *Disquisitiones Arithmeticae* appeared in 1801. It consisted of seven sections with all but the last devoted to higher arithmetic, that is, number theory. The final piece contained his construction of the regular 17-gon. Gauss summarized previous work in a systematic way, solved some of the most difficult outstanding questions, and formulated concepts and questions that set the stage for future research in the subject. He submitted a large part of the work in a memoir to the French Academy in 1800, but it was unkindly rejected, and even when published, it had only a small readership. He was hurt by the rejection, and this may explain his reluctance to publish his findings in future years.

His next work was of a completely different nature. It had long been speculated that there should be a planet in the space between Mars and Jupiter. It wasn't until 1801 that the Italian astronomer Giuseppe Piazzi discovered in the aforementioned space a new "small planet," which he named Ceres. Unfortunately, Piazzi had only been able to observe a minute portion of its orbit before it disappeared behind the Sun. An astronomer named Zach communicated Piazzi's observations to Gauss, who launched into calculations. Zach then published several predictions of where Ceres would be at different times. On December 7, 1801, Ceres appeared exactly where Zach and Gauss had predicted. Although Gauss did not reveal his methods, it was later learned that he had used his least squares approximation method.

From 1804 to 1807, Gauss exchanged a few letters at a high mathematical level with Sophie Germain, but he never visited her in France or collaborated with any of the other mathematical giants of the day. He remained isolated, as he had been since childhood, cranking out research paper after research paper in many diverse branches of mathematics and its applications. When the Duke was killed fighting for the Prussian Army in 1807, Gauss left Brunswick to accept the position of director of the Göttingen Observatory and professor of Astronomy. He remained in these offices until his death, only leaving his Observatory once in that period to attend a scientific congress in Berlin. In 1809, Gauss published his *Theoria Motus Corporum Coelestium*, a treatise that contributed greatly to the improvement of practical astronomy.

Sometime later Gauss took up the subject of geodesy, acting from 1821 to 1848 as scientific advisor to the Danish and Hanoverian Governments for the survey then in progress. While working on this survey Gauss invented the heliotrope, an instrument that worked by reflecting the Sun's rays using mirrors and a small telescope. Gauss' work in geodesies led him to develop the powerful method of conformal mapping, the kind of mapping that preserves angles. Because it is not possible to plot a three-dimensional figure perfectly onto a two-dimensional plane, it is necessary to decide, before a map is drawn, what is to be preserved and what is to be distorted.

In 1822, Gauss won the Copenhagen University Prize for his *Theoria attractionis corporum sphaeroidicorum ellipticarum homogeneorum methodus nova tracta*, which was principally concerned with potential theory. His first paper on electricity and magnetism, *Intensitas Vis Magneticae Terrestris ad Mensuram Absolutam Revocata*, was published in 1833. Gauss also wrote papers on mathematical statistics. Shortly before his death he correctly predicted that "analysis situs" [topology] would become one of the chief concerns of mathematics in the future.

From early in the century, Gauss had looked into the question of the possible existence of a non-Euclidean geometry. He discussed the matter with Farkas Bolyai and in correspondence with C.L. Gerling and H.C. Schumacher. In an 1816 book review, Gauss discussed proofs that deduced the parallel postulate from other Euclidean axioms, suggesting that he believed in the existence of non-Euclidean geometry, although he was rather vague on the subject. He confided to Schumacher that he believed his reputation would suffer if he publicly admitted that he believed in the existence of such a geometry. Nonetheless, Gauss continued his inquiry, finding more results in the "new" geometry. He considered writing up his findings, possibly to be published after his death. When he learned of Nikolai Lobachevsky's work on the subject, he praised its "genuinely geometric" character, but also related to Schumacher that he had had the same convictions for some 54 years. That means he first considered the possibility of the existence of an alternative to Euclidean geometry when he was 15. Gauss introduced the term "non-Euclidean geometry" and shares with Lobachevsky and János Bolyai credit for its creation.

Gauss was married twice, outliving both his wives. One son and a daughter survived from his first marriage to Johanna Osthoff, whom he greatly cherished. He had three children with Johanna's friend, Minna Waldeck. The second marriage seemed one of convenience rather than great love. Gauss dominated his daughters and quarreled so much with his younger sons that they moved to the United States. Minna's death in 1831 settled a domestic problem. Gauss had been invited to take a position at the University of Berlin, which Minna and her family were keen for him to accept, but he didn't like change and decided to stay at Göttingen. His daughter Therese took over the household after her mother's death and was Gauss' intimate companion during the last twenty-four years of his life. From 1850 onward, his work was mostly of a practical nature, although he did supervise G. F. B. Riemann's doctoral thesis. In 1849, Gauss celebrated his golden jubilee, and was well enough to attend the opening of the new railway sector between Hanover and Göttingen, but his health was deteriorating. He suffered from an enlarged heart and shortness of breath, and symptoms of dropsy. He died peacefully in his sleep on February 23, 1855 in his seventy-eighth year.

Quotation of the Day: "Mathematics is the queen of sciences and arithmetic [number theory] is the queen of mathematics. She often condescends to render service to astronomy and other natural sciences, but in all relations she is entitled to the first rank." – Carl Friedrich Gauss