# Évariste Galois

What follows the sad story of a brilliant mind shut down before its owner had a chance to contribute to mathematics all of which he was capable. It is often difficult to separate the facts from the legend in the account of the life of **Évariste Galois (**October 25, 1811 – May 31, 1832). In his short lifetime he gave the necessary and sufficient conditions for the solvability of polynomial equations by radicals. His work was not understood until eleven years after his death when Joseph Liouville examined his manuscript and declared that it indeed solved the problem proposed by Paolo Ruffini, Niels Abel, and others. Galois' mathematical reputation rests on less than 100 pages of mostly posthumously published memoirs.

Galois was born in Bourg La Reine, near Paris and was mortally wounded in a duel with pistols on May 30, 1832, dying the next day. His father, Nicholas-Gabriel Galois, a prominent member of the community, was elected mayor in 1815. Galois' mother, Adélaïde Marie Demante, was his only teacher until he was 12, giving her son an excellent education in Latin, Greek and rhetoric, but apparently no mathematics. Both of his parents were staunch Republicans at the time the monarchy was reestablished after the rule of Napoleon. Galois exceeded his parents in Republican zeal. He enrolled at the Lycée Louis-le-Grand in 1823 where he received several prizes and had a good record but was required to repeat a year in 1826 because he wasn't judged mature enough for the next class. At about this time Galois took his first mathematics course and from then on became absorbed with the subject. One of his mathematics teachers, Louis Richard, gave the young genius his head, allowing him to study the works of the masters on his own, including Adrien Marie Legendre's *Géométrie* and the treatises of Joseph-Louis Lagrange. Another mathematics teacher, M. Vernier, recommended that Galois work more systematically.

It is probably fair to describe Galois as headstrong and willful. Whether this was because of his genius or because his teachers and other mathematicians should have recognized his extraordinary gifts is debatable. Not every student who is "singular, bizarre, original and withdrawn" is a major mathematical talent. Galois published his first paper on continued fractions in April 1829 and in May and June submitted articles on the algebraic solution of equations to the *Académie des Sciences*. The esteemed mathematician Augustin-Louis Cauchy was appointed to referee the works. In *Men of Mathematics,* Eric Temple Bell claims that Cauchy, perhaps unable to recognize the significance of the work, or uninterested in the ideas of some unknown youth, mislaid the work, perhaps intentionally. One must take into account, that even while acknowledging Cauchy's mathematical genius Bell vehemently disliked the man.

At age 16, Galois took the examination to enter the prestigious École Polytechnique, but failed. Bell and others blame the failure, at least in part, on the arrogance of a young man who did not suffer fools well, and fools were what he considered his examiners to be. They put questions to him about mathematics he considered trivial. He wished to demonstrate his abilities by describing the problems he had studied and the mathematics he had invented to solve them. His examiners were not interested, insisting instead that he answer their questions. His halfhearted attempts to do so were not impressive. It has been reported that Galois was never very good at communicating his thoughts, perhaps because to him things were too clear to need elaboration. At 18, Galois reapplied to the Polytechnique, and once again the examination went badly. He was no more tolerant of the new group of examiners and their questions than he had been with their predecessors. He was disgusted that they did not wish to hear about the gems of mathematics he had to share. According to Bell, in frustration he picked up an eraser and bounced it off the head of the chief examiner. There would be no further attempts to gain acceptance to the Polytechnique. Besides his unhappiness with the questions put to him, it is likely that he was understandably distraught because his father had hanged himself. By Bell's account, "the elder Galois was a target for the clerical intrigues of the time." A young priest composed a set of filthy verses, which he signed with Mayor Galois' name. The priest then circulated the verses, directed against the Galois' relatives, among the citizens of Bourg-la-Reine. Galois' father developed a persecution conflict, which made it impossible for him to live.

Galois failure to gain admission to the Polytechnique meant he would not be able to commune with the likes of Lagrange, Laplace, Fourier, Cauchy and Poisson, perhaps dooming his prospects of being recognized as an outstanding mathematician or of securing a prominent position in the mathematical establishment. Instead he was reduced to enrolling at the École Normale to prepare to be an ordinary mathematics teacher. Things, however, did not go much better at the Normale. In addition to his mathematical researches, the ardent Republican and hothead became passionately involved in the political upheaval of the time. He was arrested twice, once for making a toast that seemed to suggest he was threatening the king with a dagger. His lawyer was able to get the youth freed at his trial by asserting Galois had been misquoted due to the noise at the gathering. On Bastille Day, July 14, 1831, Galois was arrested for being in the uniform of the outlawed Artillery of the National Guard, carrying a loaded rifle, several pistols and a dagger. While in prison he received the bad news that referee Simon Poisson and the Academy had rejected his memoir (the third version of the memoir that Cauchy may or may not have lost). Poisson confessed that he had not comprehended Galois' ideas, because they were neither sufficiently clear nor developed far enough to judge their rigor. Yet he did not dismiss the work as valueless, concluding his report with the statement:

> "The author claims that the proposition which is the subject of his memoir is part of a general theory rich in application. Often different parts of a theory are mutually clarifying,

> and it is easier to understand them together than in isolation. One should rather wait for the author to publish his work in entirety before forming an opinion."

Galois took no comfort from Poisson's suggestion that a more complete work might be viewed with greater favor. On March 16, 1832 Galois and other prisoners were transferred to the pension Sieur Faultrier to prevent them from being exposed to a cholera epidemic sweeping through Paris. Little is known of the events between this time and his release from prison on April 29, except he may have fallen in love with Stephanie-Felicie Poterin du Motel, daughter of a resident physician at the hospital. Apparently the love was unrequited, as in a May 25 letter to a friend Galois alludes to a broken heart. In less than a month after being released from prison, Galois was challenged to a duel. The events of the night before the duel have been the source of much speculation. Here's how Bell's delicious prose set the scene:

> "All night long he had spent the fleeting hours feverishly dashing off his scientific last will and testament, writing against time to glean a few of the great things in his teeming mind before the death which he saw could overtake him. Time after time he broke off to scribble in the margin 'I have not time; I have not time,' and passed on to the next frantically scrawled outline. What he wrote in those desperate last hours before dawn will keep generations of mathematicians busy for hundreds of years."

In fact the evening before his duel Galois wrote to Republican friends. In one he claims to be "the victim of an infamous coquette." Bell suggests that she might have been some low prostitute. Leopold Infeld, in his fictional biography, *Whom the Gods Love: The Story of Évariste Galois* (1948) she was a female provocateur who set up Galois to be killed in a duel with a police agent, thus ridding France of a perceived dangerous radical. An account of the duel in a Lyon newspaper article published several days after it occurred claimed that love was the cause of the combat, and that Galois was killed by an old unidentified friend. It would make marvelous drama, but bad mathematics to believe that it was the night before the duel, in the state he must have been, that he created his marvelous theory of equations. Galois had already written memoirs on his use of groups in his work in the theory of equations. In a long letter to Auguste Chevalier, written that night, Galois described his theory and the contents of the memoir rejected by Poisson.

As the legend, goes the next morning he appeared at the dueling grounds, was fatally wounded and abandoned by his opponent and even his own seconds. He was discovered by a peasant and taken to a hospital. When his younger brother Alfred arrived, Évariste told him, "Don't cry, I need all my courage to die at twenty." He died the day after the duel and was buried in an unmarked common grave. Alfred and Chevalier copied his mathematical papers and sent them to Gauss, Jacobi and others as Évariste had requested in his last letters. And then his legend began. What seems a far more balanced account of the Galois story than provided by either Bell or Infeld is found in Tony Rothman's 1989 book *Science à la Mode.* In his essay, entitled "Genius and Biographers: The Fictionalization of Évariste Galois," Rothman reviews the tales told about Galois, questioning and analyzing those that seem romantic inventions or contrived intrigues.

As so much about Galois' life and death is in question it is fitting to end this account with something certain, his mathematical legacy. The solution of quadratic equations goes back to ancient people, including the Babylonians, Chinese, and Hindus. Girolamo Cardano published formulas in 1545 for solutions of the cubic and quartic equations, after Niccolo Tartaglia and Ludovico Ferrari discovered them a few years earlier. Mathematicians were unable to find formulas involving radicals for solving quintic degree equations. In 1796 Ruffini attempted to prove this was impossible, but his efforts were not wholly successful. In 1824 Niels Abel gave an essentially correct proof. Galois was unaware of Abel's work when he began his own investigation. This is perhaps just as well, as the solution of the problem of the quintic is far less important than the theory Galois invented for its solution. He treated the problem of solving the quintic and higher order equations, using only the four basic operations and the extraction of roots, by studying groups of permutations. By abstracting equations and looking for their solutions in terms of groups, he was able to determine their solvability. He was not concerned with finding particular solutions, but in the more fundamental question of determining 'a priori' whether or not the solution existed. He proved that no general method using purely algebraic formulas could be found to solve equations of degree five or higher. His results became a major factor in the evolution from classical to modern algebra, from the solving of equations to the study of systems.

**Quotation of the Day**: "In my life I have dared to advance propositions about which I was not sure. But all have written down here has been clear in my head for over a year, and it would not be in my interest to leave myself open to the suspicion that I announce theorems of which I do not have complete proofs. Make a public request of Jacobi or Gauss to give their opinions not as to the truth but to the importance of these theorems. After that, I hope some men will find it profitable to sort out this mess." – Évariste Galois, Letter to Auguste Chevalier, May 29, 1832.