Eudoxus

In Greek mythology the worship of the god Zeus involved a cosmology built on the assumption of a stationary earth. Ancient Greek philosophers taught that the sky was a solid sphere, which rotated, and held all the stars in space. The giant Atlas was one of the Titans who revolted against Zeus. As a punishment he was condemned to carry the vault of heaven on his shoulders. The philosopher Plato believed the heavens to be more perfect than the earth and urged astronomers to describe celestial motions in terms of the circle, the most perfect of all geometric shapes. In the 4^{th} century BCE, Greek mathematician, astronomer, geographer, physician and philosopher **Eudoxus** (c. 408 – 355 BCE) took up Plato's challenge. Eudoxus advanced the theory that the planets were not circling in the sky under the control of gods and that the solution of cosmic "mysteries" did not depend upon divine revelation, but could be discovered by rational thought.

The son of Aischines, Eudoxus was born in Cnidus, Asia Minor (now part of Turkey). He grew up in extreme poverty but was taught by Archytas, then head of the Pythagoreans at Tarentum, where Eudoxus learned geometry, number theory, and music theory. He visited Sicily, where he studied medicine with Philiston before making his first visit to Athens. Eudoxus could not afford to live near Plato's Academy, so he walked back and forth each day from Piraeus, a port of Athens, where he could live cheaply. After a couple of months he traveled to Egypt, where he learned astronomy from the priests at Heliopolis. He then moved to Cyzicus on the sea of Marmora where he established a school and had many students.

Around 368 BCE Eudoxus and some of his followers went to Athens to be with Plato. The latter recognized his visitor's enormous abilities, but when it became apparent that Eudoxus was mathematically superior, the two had a falling out. This, combined with Eudoxus' unpopularity because he was a foreigner, caused him to leave Athens and return to Cnidus, where he was met with acclaim and was given an important position in the government. He continued his scholarly work, writing and lecturing on theology, astronomy and meteorology. He died while on a journey to Egypt in 355 BCE.

In an attempt to explain planetary motion, Eudoxus became the first individual to offer a geometrical model of the heavens. Pythagoras was the first astronomer to suggest the idea of a geocentric universe, which consisted of concentric spheres into which the Sun, the Moon, and the five wanderers, the known planets, Mercury, Venus, Mars, Jupiter and Saturn, were embedded. Unfortunately the theory was not supported by observation, which might have revealed that the planetary motions did not act in accordance with the patterns they should have had they been following perfectly circular orbits around the earth. Eudoxus proposed a nested system of "spheres within spheres," which introduced the study of spherics (mathematical astronomy) to Greece. In his system, the spherical earth is at rest at the center. Around this center rotate 27 concentric spheres. The interior spheres carry the fixed stars, and the others account for the sun, moon, and the five planets. Each planet is attached to four spheres, the sun and the moon, three each. These spheres rotate at different speeds about different axes.

Figure 5.9 schematically illustrates the spheres for Mars. The outer sphere rotates once per Earth-day about an axis passing through the celestial poles. The second sphere rotates at a slower rate representing the motion of Mars through the Zodiac, carrying it around the ecliptic once per Mars-year. Its axis is attached to the first sphere and swings as the outer sphere revolves in its daily rotation. The axis of the third sphere is attached to the second sphere and swings as the second one turns. Similarly the axis of the fourth sphere, which carried Mars, is attached to the third sphere. The rotations of the third and fourth spheres have nearly opposite motions and give Mars its retrograde (moving backward) motion.

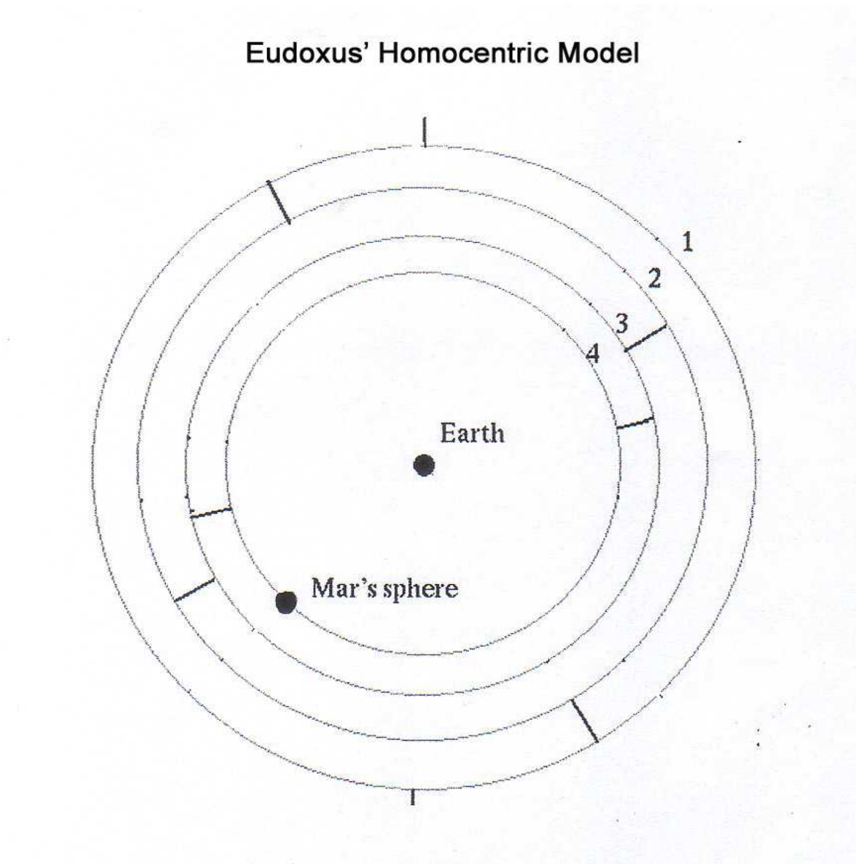

Figure 5.9

Eudoxus' work in astronomy was surpassed by his mathematics. He placed the doctrine of irrational numbers upon a thoroughly sound basis by developing a theory of proportion that ended the crises caused by the discovery of the existence of incommensurable quantities, which could not be accounted for in Pythagorean mathematics. It was so well done, that his theory influenced the work of the nineteenth century arithmetical reconstructions of Richard Dedekind and Karl Weierstrass. Eudoxus introduced the notion of a magnitude that is not a number but represents entities that varied continuously such as line segments, angles, areas, volumes, and time. Magnitudes differed from numbers, which were discrete, that is, they jumped from one value to another, as from 5 to 6. No quantitative values were assigned to magnitudes. Eudoxus defined a ratio of magnitudes and a proportion to cover the cases of commensurable and incommensurable (rational and irrational) ratios. This enabled him to avoid troublesome infinitesimals. Eudoxus first applied the “axiom of continuity,” which was later expanded by Archimedes. It is the basis of both integral and differential calculus and centuries later Isaac Newton and Gottfried Leibniz based their work on this assumption. In addition, Eudoxus developed the method of exhaustion, a process close to the limiting concept of calculus, later used to find areas and volumes of curvilinear figures.

In geometry, Eudoxus established principles that laid the foundation for Euclid’s work in his *Elements*. He was among the first to investigate the properties of a curve called the *hippopede*, which literally means “foot of a horse” and is also known as the “horse-fetter.” In polar coordinates the equation of the curve is $r^2 = 1 - a \sin^2 \varphi$, and when $a = 1$, it resembles the present-day symbol for infinity. When $a = 2$, the curve is the lemniscate of Bernoulli. According to Eutocius, Eudoxus solved the Delian problem (duplication of the cube) using a curve [Figure 5.10] known as the “Kampyle of Eudoxus,” which is given by the equation $y^2 = x^4 - x^2$. If a circle is constructed passing through **O** with radius $\sqrt{1/2}$, intersecting the kampyle at **P**, the length of **OP** is $\sqrt[3]{2}$. Perhaps most importantly, Eudoxus had a profound influence on the establishment of deductive organization of proof based on explicit axioms.

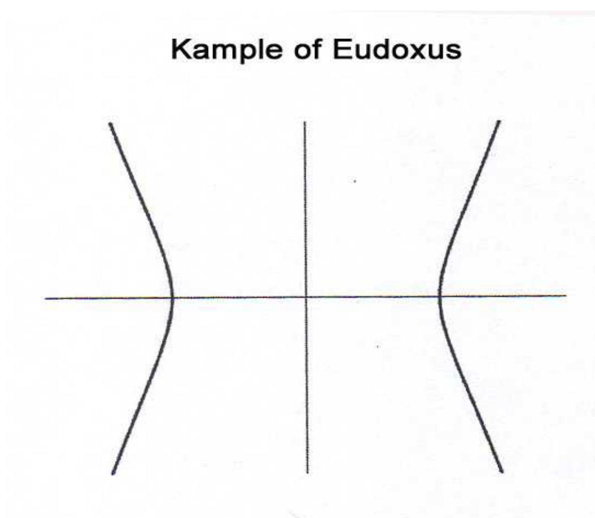

Figure 5.10

Quotation of the Day: “Willingly would I burn to death like Phaeton, were this the price for reaching the sun and learning its shape, its size and its substance.” – Eudoxus