

## Euclid

**Euclid** (c. 325 BCE – c. 265 CE) was of the best selling mathematics textbook of all time. *The Elements* has introduced millions to the powers of logic and language. As a struggling lawyer at age forty, Abraham Lincoln reflected:



“I said, ‘Lincoln, you can never make a lawyer if you do not understand what *demonstrate* means’; and I left my situation in Springfield, went home to my father’s house, and stayed there till I could give any proposition in the six books of Euclid at sight. I then found out what ‘demonstrate’, means, and went back to my law studies.”

Euclid did not call his work ‘geometry;’ the word didn’t appear any place in his thirteen books, possibly because at the time geometry referred to land measure. Euclid’s *Elements*, written some time around 300 BCE, was in use for some twenty-three centuries. His name is better known than that of any other mathematician who ever lived. Today’s high school geometry students do not use the historic *Elements*, and it’s probably a good thing. In his book *Dots and Lines*, Richard J. Trudeau notes what isn’t and what is found in *The Elements*:

“People who read *The Elements* for the first time often get a feeling that things are missing: it has no preface or introduction, no statement of objectives, and it offers no motivation or commentary. Most strikingly, there is no mention of the scientific and technological uses to which many of the theorems can be put, nor any warning that large sections of the work will have no practical use at all.... The theorems are included for their own sake, because they are interesting in themselves. This attitude of self-sufficiency is the hallmark of pure mathematics.”

Euclid was the first major scholar at the Library of Alexandria in Egypt, founded by King Ptolemy, the illegitimate son and successor of Alexander the Great. The Library - sometimes called the Museum, coined because the institution was dedicated to the seven muses - was a center of learning as well as a repository of knowledge. Estimates of the number of its scrolls when Alexandria was the cultural center of the Hellenistic range from 200,000 to 700,000. It represented the world's knowledge at that time. Ptolemy's successors went to great lengths to obtain scrolls that could be copied and become part of the collection. Desiring a Greek translation of the Hebrew Bible, Ptolemy II imprisoned seventy Jewish scholars in cells on the island of Pharos until they completed the work. Reportedly, Ptolemy III wrote to all the world's rulers, asking to borrow their books, and the ones that were sent to him, he kept.

Of the man, little is known. Some scholars have explored the idea that Euclid was a leader of a band of mathematicians who working together produced the *Elements*, making him a sort of early day Nicolas Bourbaki. While there isn't any evidence to refute this there isn't any credible reason to believe it either. There is even less reason to believe another suggestion that Euclid was not a historical character and that the group of mathematicians who prepared the *Elements* named its author in honor of the philosopher Euclid of Megara, who lived 100 years earlier. The important thing is what *The Elements* has meant to mathematics. Apparently its author patiently collected all the geometrical facts known in his day, arranged the theorems in a logical order, improved their proofs where necessary, and added theorems from his own investigations. In his proofs, he justified the steps by references to previously proved results. His organization and logical demonstrations remained a model for mathematical reasoning for some 2000 years. As Alfred Hooper wrote in *Makers of Mathematics*,

“He was the master mind that was able to collect all the muddled, confused pieces of a vast mathematical jigsaw puzzle and put them together in such a way that a clear and beautiful picture suddenly emerged from what had been a welter of odds and ends of mathematical knowledge. “

Euclid stated 23 definitions, five geometric postulates, five additional axioms, which he called common notions, and proved 465 theorems, all the geometrical knowledge known in his day. Each of the thirteen books of the *Elements* was written on a separate roll of parchment. For twenty centuries the first six books were the student's usual introduction to plane geometry. Books seven to nine dealt with number theory, book ten treated the theory of irrational numbers, and books eleven to thirteen featured solid geometry. Euclid did not claim that all of the theorems and proofs found in *The Elements* were original with him, although some surely were. However the book's format was all due to him. Euclid wasn't the first to have made a compilation of geometry. Hippocrates of Chios is said to have written *Elements of Geometry* in the mid fifth century BCE. Theaetetus of Athens, who lived from about 417 BCE to 369 BCE made many important contributions to mathematics, which are described in Book X and Book XIII of Euclid's *Elements*.

The principal translator of Euclid into Latin was Anicius Manlius Severinus Boethius in the 6<sup>th</sup> century CE. He gave only the definitions and theorems of Book I with no proofs. He did include a large number of practical applications of the selected propositions. His *Geometry* and *Arithmetic* were the basis for most of the mathematical teaching of the early medieval times in Europe. Late in the 8<sup>th</sup> century, an Arab caliph came into possession of a copy of the *Elements* in the original Greek. Several Arabic translations were made from it, including one at the time Harun al Rashid, the caliph known to posterity from Scheherazade's *Arabian Nights Tales*. In 1120 Adelard of Bath made a Latin translation from an Arabic version. Several other Latin translations were made, including the first important mathematical book to be printed (1482). It contains beautiful figures engraved on its wide margins. Since this first printed version, over a thousand editions of the work have been published. The first English translation of the *Elements* appeared in 1570.

In addition to the *Elements*, Euclid is known to have written an elementary treatise on conic sections, a

book on surface loci, a collection of geometrical fallacies, and a treatise on porisms (that is, propositions derived from others). All of these are lost. Among his existing works is a book on optics, treated geometrically, which consists of 61 propositions based on 12 assumptions. Euclid also wrote the *Phaenomena*, a treatise consisting of 18 propositions, concerning the basic geometry of the celestial sphere. His *Data* contains 94 propositions that demonstrate that when certain aspects of a figure are given, then other aspects of the figure are also given. These propositions may be considered elementary exercises in analysis, supplementing the theorems and problems found in *The Elements*. *On Divisions of Figures* consists of 36 propositions concerning the division of various figures into two or more equal parts or parts in given ratios.

There was a time, a very long time, when Euclid's *Elements* was considered to be a sort of secular version of the Bible in that it contained absolute truths. For instance, in *Prolegomena to any Future Metaphysics*, Immanuel Kant wrote, "There is no single book about metaphysics like we have in mathematics. If you want to know what mathematics is, just look at Euclid's *Elements*." When non-Euclidean geometries were created, *The Elements* was downgraded to the stature of relative proof. At least Euclid had quite a run. Whether he believed he was dealing with absolute truth, relative truth or any kind of truth, we'll never know. He certainly didn't compile the thirteen books with any anticipation of them having any practical use. As Alfred Hooper suggested, he seemed to be one who had the many pieces of a jigsaw puzzle spread out around him, determined to fit them together to reveal a beautiful whole. He wasn't completely successful in this endeavor, for his work was not logically perfect and not all of his proofs can be accepted as such any more, but it seems picky to criticize Euclid for not knowing twenty-three hundred years ago what is known now. In the 20<sup>th</sup> century, David Hilbert completed Euclid's geometry by increasing the number of postulates to 20 and proved the resulting system was complete and consistent.

## **Quotation of the Day:**

When first the shaft into his vision shone  
Of light anatomized! Euclid alone  
Has looked on Beauty bare. Fortunate they  
Who, through once only and then but far away,  
Have heard her massive sandal set on stone.

- Edna St. Vincent Millay