Eratosthenes

Greek scholar, geographer and astronomer **Eratosthenes** (c.76 BCE – 194 BCE) was

born in Cyrene, North Africa (now Shahhat, Libya). Neither the exact date of his birth

or of his death in Alexandria, Egypt is known. He was the first systematic geographer

of antiquity, the director of the library at Alexandria, a mathematician, poet,

philosopher, historian, philologist, chronologist, and reputedly one of the most learned

men of antiquity. He was a student of Callimachus and studied at Plato's academy in

Athens. He was invited to Alexandria by Ptolemy Euergetes to tutor his son Philopater.

The library at Alexandria had been planned by Ptolemy I Soter and came to fruition

during the reign of his son Ptolemy II Philadelphus. The original holdings of the library were based on copies of the works

in Aristotle's library. Eratosthenes served as the director of the library for several decades, working there until he lost his

sight to ophthalmia in old age. Unable to read due to the affliction he was unwilling to live and starved himself to death.

Eratosthenes' chief scientific interests were astronomy and geodesy. He constructed various astronomical instruments used

for several centuries at the university. He suggested that a leap day be added every fourth year to the calendar and he

attempted to construct an accurate chronology of the world by giving dates of literary and political events from the time of

the siege of Troy. Eratosthenes determined the obliquity of the ecliptic, measuring the tilt of the earth with great accuracy to

obtain the value 23º 51´ 20´´. He prepared a star map containing 675 stars and wrote a poem called "Hermes," which

described the fundamentals of astronomy in verse. Eratosthenes collected the available geographical knowledge and made

numerous calculations of distances on the earth between significant places. He wrote his *Geography*, in which he

incorporated the methods and the results of his measurements and calculations. He also made a map of the world. In

addition he sketched, rather accurately, the route of the Nile to Khartoum, showing the two Ethiopian tributaries, and

suggested that the source of the Nile were lakes. Eratosthenes' map of the world has not survived, only descriptions of it as

recorded by others who viewed the original. Reconstructions of the map according to Eratosthenes have been made based

on these descriptions.

Of Eratosthenes's work in mathematics there are two extant examples: one is a description of an instrument to duplicate a

cube, and the other is a rule he gave for constructing a table of prime numbers. The "sieve of Eratosthenes" works as

follows: list the whole numbers from 1 onward [Figure 5.4]; since 1 is not a prime by definition, cross it out; then every

second number from the prime 2 is a multiple of 2 and may be crossed out; every third number from the prime 3 is a

multiple of 3 and may be crossed out; every fifth number from the prime 5 is a multiple of 5 and can be crossed out; and so

forth. It has been estimated that working manually in this way would require some 300 hours to find the primes in the

numbers from 1 to 1,000,000. The task of determining whether a particular number is a prime is shortened by observing

that if a number is the product of two factors, one must be less and the other greater than the square root of the number,

unless the number is a square of a prime, in which case the two factors are equal. Hence every composite number must be

divisible by a prime that is not greater than its square root.

Fig. 5.4

Eratosthenes' most famous calculation is the circumference of the earth. At noon of the summer solstice, he observed the

sun to be practically overhead at the city of Syene (now Aswan). This was confirmed by observing that the sun shone

directly down a well there. Eratosthenes knew the earth was a sphere and correctly reasoned that if he could determine the elevation of the noon sun at some other location of the first day of the summer, and if the distance between these two locations was known, he could compute the circumference of the earth as a simple ratio. On the first day of summer Eratosthenes placed an upright rod in the ground to form a simple sundial (gnomon) at Alexandria and took a reading of the noon sun. At the same time an upright rod at Syene would cast no shadow. In Alexandria, which is within 3° on the same meridian as Syene but north of it, the angle between the overhead direction for the location (AB in Figure 5.5) and the direction of the sun (AD in the figure) was observed to be 1/50 of 360° (about 7.2°). The sun is so far from the earth that SE and AD may be considered parallel. Thus SA is 1/50 of the circumference of the earth. Eratosthenes estimated the distance from Alexandria to Syene by using the fact that camel trains, which traveled 100 stadia a day, took 50 days to reach Syene. Hence the distance is 5000 stadia.
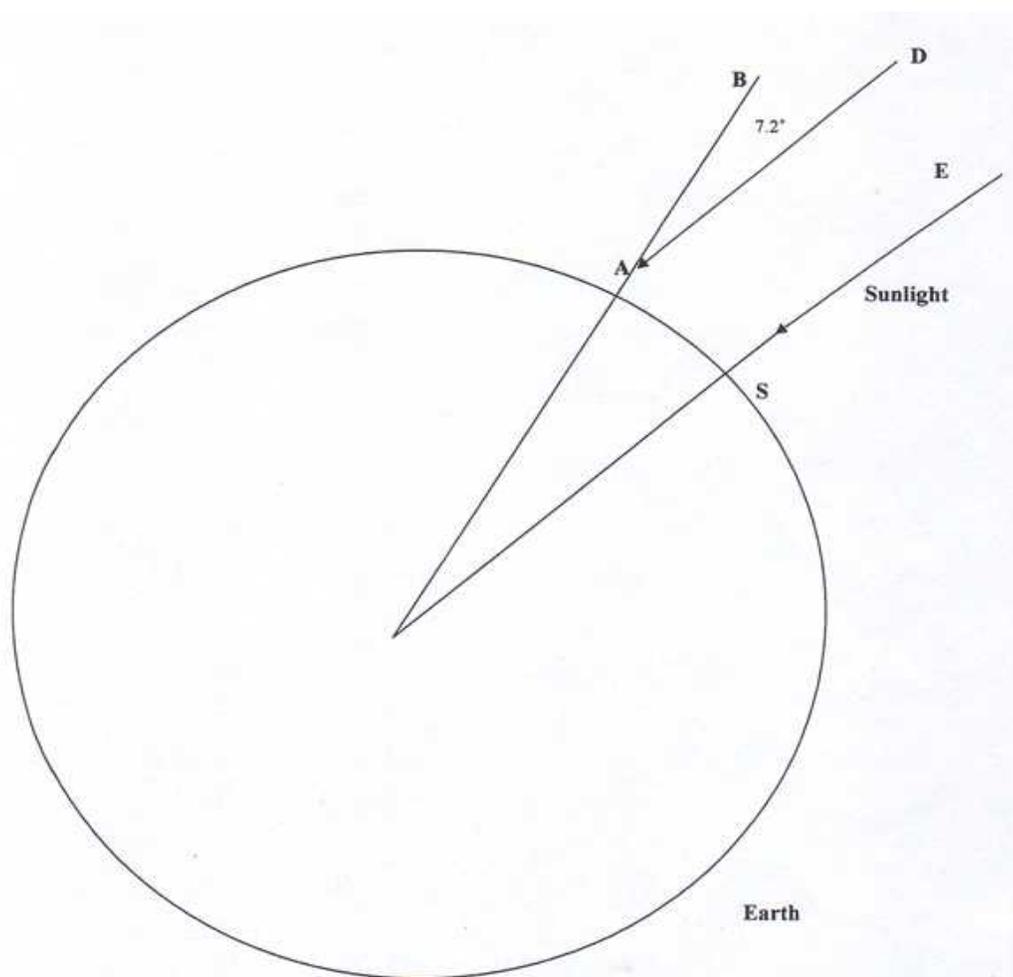

Figure 5.5

$$\frac{\text{Circumference of the Earth}}{\text{Distance from Alexandria to Syene}} = \frac{360°}{\text{angle of suns ray at Alexandria}}$$

Circumference = 5000 x 360/7.2 = 250,000 stadia.

Using the Egyptian stadia of about 173.6 yards, Eratosthenes' result is 24,662 miles, which is very close to the actual circumference of the earth, as it was measured centuries later.

Eratosthenes was not the only Greek who attempted to measure the circumference of the earth. About a century later, Posidonius used the star Canopus as his light source and the cities of Alexandria and Rhodes as his points of reference on earth. Unfortunately, he miscalculated the distance between Alexandria and Rhodes, resulting in an estimation of the circumference of the Earth that was much too small. Ptolemy recorded this incorrect figure in his geography. The inaccuracy convinced explorers such as Christopher Columbus that sailing west would result in a quicker journey to India. One can but speculate what might have occurred had Ptolemy used Eratosthenes' calculation rather than that of Posidonius.

Quotation of the Day: "[Eratosthenes] was… recognized by his contemporaries as a man of great distinction in all branches of knowledge, though in each subject he just fell short of the highest place. On the latter ground he was called Beta, and another nickname applied to him, Pentathlos, has the same implication, representing as it does an all-round athlete who was not the first runner or wrestler but took the second prize in these contests as well as others." – Thomas L. Heath