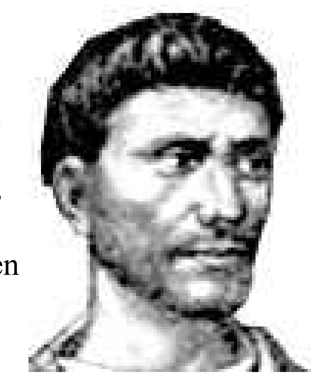
Diophantus of Alexandria

Diophantus of Alexandria played a major role in the development of algebra and was a considerable influence on later number theorists. Diophantine analysis, which is closely related to algebraic geometry, has experienced a resurgence of interest in the past half century. Diophantus worked before the introduction of modern algebraic notation, but he moved from rhetorical algebra to syncopated algebra, where abbreviations are used. Prior to Diophantus' time the steps in solving a problem were written in words and complete sentences, like a piece of prose, or a philosophical argument. Diophantus employed a symbol to represent the unknown quantity in his equations, but as he had only one symbol he could not use more than one unknown at a time. His work, while not a system of symbols, was nevertheless an important step in the right direction. François Viète, influenced by Napier, Descartes and John Wallis, introduced symbolic algebra into Europe in the 16th century when he used letters to represent both constants and variables.



Claims that Diophantus lived from about 200 to 284 and spent time at Alexandria are based on detective work in finding clues to the times he flourished in his and others' writings. Theon of Alexandria quoted Diophantus in 365 and his work was the subject of a commentary written by Theon's daughter Hypatia at the beginning of the 5th century, which unfortunately is lost. The most details concerning Diophantus' life are found in the *Greek Anthology*, compiled by Metrodorus around 500. Diophantus is often referred to as the "father of algebra," but this is stretching things as many of his methods for solving linear and quadratic equations can be traced back to the Babylonians. He is best known for his *Arithmetica*, a work on the solution of algebraic equations and on the theory of numbers. Only 6 of its original 13 books have survived.

Diophantus' earliest work was probably his short essay on polygonal numbers, containing ten propositions in which he employed the classical method in which numbers are represented by line segments. In the *Arithmetica*, Diophantus showed how to solve linear equations and 3 different types of quadratic equations, but considered only rational solutions. The extant portion of the *Arithmetica* consists of the solution of 130 problems, involving both determinate and indeterminate equations. The method for solving the latter is now known as Diophantine analysis. An equation is indeterminate if it has more than one variable. An example is the problem of finding Pythagorean triplets, that is, integers x, y, and z that satisfy the equation $x^2 + y^2 = z^2$. Historically, the problem of solving "Diophantine equations" has been to find expressions that show the relationship of the integral values of x and y that satisfy an indeterminate equation "$ax + by = c$" where the coefficients a, b, and c are integers. For example, $x + 2y = 4$ has an unlimited number of solutions.

The following are examples of the problems solved in the *Arithmetica*:

1. Find four numbers, the sum of every arrangement three at a time being given; say 22, 24, 27, and 20. The following solution is based on Diophantus' directions, except of course for the use of modern notation. Let x be the sum of all four numbers; hence the numbers are $x - 22$, $x - 24$, $x - 27$, and $x - 20$. Therefore

   $$x = (x - 22) + (x - 24) + (x - 27) + (x - 20)$$
   $$x = 4x - 93;\ 3x = 93,\ x = 31$$
   and thus the four numbers are 9, 7, 4, and 11.

2. Divide a number, such as 13, which is the sum of two squares 4 and 9, into two other squares. Diophantus says that since the given squares are 2^2 and 3^2 he will take $(x + 2)^2$ and $(mx - 3)^2$ as the required squares, and will assume $m = 2$. Therefore,

   $$\begin{aligned} (x+2)^2 + (2x-3)^2 &= 13 \\ x^2 + 4x + 4 + 4x^2 - 12x + 9 &= 13 \\ 5x^2 - 8x + 13 &= 13 \\ x(5x - 8) &= 0 \end{aligned}$$

$$x = 8/5$$

Therefore the required squares are 324/25 and 1/25. Diophantus would not have considered the solution $x = 0$, as zero was not then in use.

In the *Arithmetica* Diophantus states certain theorems without proofs. It is believed that his proofs were found in his now lost *Porisms*, a collection of lemmas that probably were not in a separate book but were part of the *Arithmetica*. One of his propositions states that the difference of the cubes of two numbers can always be expressed as the sum of the cubes of two other numbers; that no number of the form $4n - 1$ can be expressed as the sum of two squares; and that no number of the form $8n - 1$ can be expressed as the sum of three squares. Claude de Bachet's 1621 Latin translation of Diophantus' *Arithmetica* is the work in which Pierre de Fermat wrote his famous comment that became known as Fermat's Last Theorem. (See the Fermat entry.)

Quotation of the Day: "This tomb holds Diophantus. Ah, what a marvel! And the tomb tells scientifically the measure of his life. God vouchsafed that he should be a boy for the sixth part of his life: when a twelfth was added, his cheeks acquired a beard; He kindled for him the light of marriage after a seventh, and in the fifth year after his marriage He granted him a son. Alas! Late-begotten and miserable child, when he had reached the measure of half his father's life, the chill grave took him. After consoling his grief by this science of numbers for four years, he reached the end of his life." – Metrodorus