GIRARD DESARGUES

There is little reliable information about the personal life of **Girard (or Gérard) Desargues** (February 21, 1591 – September, 1661), although he apparently came from a very wealthy Lyon family. His father was a royal notary. By 1626 Desargues was in Paris, and took part as an engineer at the siege of La Rochelle (1628). He later became a technical advisor to Cardinal de Richelieu and the French government. By profession a military engineer and architect, Desargues was a member of the Paris mathematical circle of Marin Mersenne. The group included René Descartes, Etienne Pascal, and his son Blaise Pascal, all of whom thought highly of Desargues' work and abilities. Each of Desargues' colleagues made considerable use of the theorems he proposed.

In 1636, Desargues derived his famous “perspective theorem,” which states,”When two triangles are in perspective, the points where the corresponding sides meet are collinear.” The theorem first appeared in print in 1648 in an appendix to a work on perspective by Desargues' friend Abraham Bosse. In this source is also found Desargues' proof of the invariance of cross ratio under projection. Desargues' Theorem states that if A, B, C and A′, B′, C′ are two triangles in the (projective) plane, the lines AA′, BB′, CC′ intersect in a single point O if and only if the intersections of corresponding sides AB, A′B′ in D; AC, A′C′ in E; and BC, B′C′ in F lie on a single line [Figure 2.1]. Each of the ten lines in the figure are so arranged that each contains three of the ten points and each of the ten points lie on three of the ten lines. The proof of Desargues' theorem in the plane is readily deduced as a corollary of the theorem in 3-space. Proofs of both can be found in any projective geometry book.

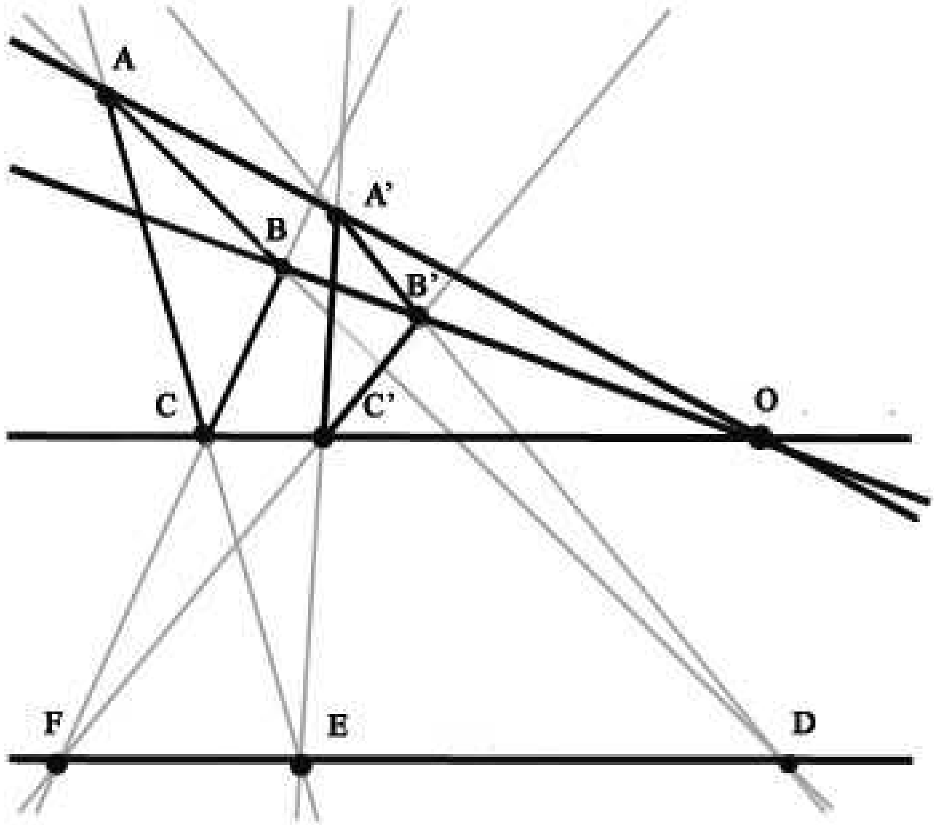

Figure 2.1

Desargues also was concerned with improving the education and technique of artists, engineers, and stonecutters. He wrote on subjects such as the cutting of stones for use in buildings, and on sundials. He also composed a small handbook of musical composition. Desargues was familiar with Apollonius' work with conics and felt he could introduce better methods for proving theorems about conics. He began by organizing a number of theorems and their proofs that he described in letters, handbills, and free public lectures. In 1636 he wrote a small pamphlet on perspective. Desargues' most important work *Brouillon projet d'une atteinte aux événemens des rencontres d'une cône avec un plan* (Proposed Draft for an essay on the results of taking plane sections of a cone) was printed in 1639. In it Desargues presented innovations in projective geometry applied to the theory of conic sections. With this work he founded the use of projective methods in geometry, inspired by the theory of perspective in art.

Desargues introduced the notions of the opposite ends of a straight line being regarded as coincident, parallel lines meeting at a point of infinity and regarding a straight line as circle whose center is at infinity. Desargues made the assumption that all the points at infinity lie on one line, which corresponds to the horizon line or vanishing line of the image of a projection. The addition of a new point on each line didn't contradict any of the axioms of Euclid but it did require some change in wording. A tangent was then defined as the limiting case of a secant, and an asymptote as a tangent at infinity. Desargues showed that conics could be discussed in terms of properties that are invariant under projection. Johann Kepler had introduced the point at infinity to parallel lines in 1604, but for a different reason, so than he might give a definition of continuity. Desargues also studied a bundle of planes through a point, finite or infinite.

Only about fifty copies of Desargues' small book were printed and distributed among his friends. Shortly thereafter all copies disappeared until a transcription made by Philippe de la Hire was found in 1845. About 1950 Pierre Moisy found an original copy of the work in the Bibliothèque Nationale in Paris. Desargues' book is difficult to read because he introduced new terminology, for instance calling a straight line a "palm." When points were marked out on the line he called it a "trunk." He called a cylindrical solid a "roller" and uses the phrase "border of a roll cut" for a conic section. His purpose in using these unusual words was in the hope of establishing clarity by avoiding confusion with familiar terms.

With the exception of his friends, who were enthusiastic about his work, most contemporaries thought he was crazy. Fermat regarded Desargues as the real founder of the theory of conic sections. Unlike Apollonius who treated each type of conic separately, Desargues offered a unified approach to the several types of conics through projection and section. However, soon thereafter, projective geometry fell into oblivion, chiefly because the analytical geometry of Descartes was viewed as a much more powerful tool to proof or discovery. Projective geometry would not be taken up again until the late 19th century, when the pupils of Gaspard Monge reinvented it. At that time it was derived from descriptive geometry, which had much in common with perspective.

Painter Laurent de la Hyre, a friend of Desargues employed the latter's ideas of perspective in his *Allegory of Geometry* (1649), which was one of the pieces of art commissioned by Gédeon Tallemant to decorate the gallery of his Hotel in the Marais quarter of Paris. Others in the series of Allegories of the Liberal Arts were: Music, Grammar, Mathematics, Astronomy, and Dialectics and Rhetoric. La Hyre portrayed Geometry as a young woman who in her right hand holds a sheet of paper containing the Golden Section and three classic Euclidean proofs. In her left hand she holds a compass and a right angle edge. The background of the painting contains a grid displaying the Cartesian coordinates and symbolic objects including a globe, a snake, a sarcophagus, and a sphinx.

Quotation of the Day: "I freely confess that I never had a taste for study or research either in physics or geometry except in so far as they could serve as a means of arriving at some sort of knowledge of the proximate causes … for the good and convenience of life, in maintaining health, in the practice of some art … having observed that a good part of the arts is based on geometry, among others the cutting of stones in architecture, that of sundials, that of perspective in particular." – Girard Desargues