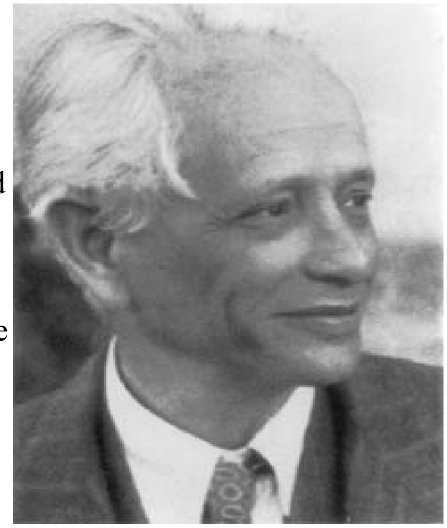
Max Dehn



German-born mathematician **Max Wilhelm Dehn** (November 13, 1878 – June 27, 1952), working jointly with Poul Heegaard, provided one of the first systematic expositions of combinatorial topology. In 1910 he published a paper on three-dimensional topology in which he introduced *Dehn surgery*, used to construct homology spheres. His work was mainly concerned with geometric properties of polyhedra and geometric group theory had its origins in Dehn's work, in which he first formulated the phrase and introduced isomorphism problems for infinite groups. He proved an important theorem on topological manifolds, known as Dehn's lemma, and also wrote on statics, projective planes and the history of mathematics.

Dehn was born in Hamburg, one of eight children of a physician. The family were secularized Jews who, according to Max's son Helmut, "lived by principles that some … would call 'good Christian'", and did not think of themselves as Jews until the Nazis came to power. After graduating from the Gymnasium in Hamburg, Dehn first went to Freiburg and later Göttingen. There, as a student of David Hilbert, he received his doctorate in 1900, for a dissertation that established Legendre's Theorem that the Archimedean postulate is essential in order to prove that the sum of the angles of a triangle does not exceed 180 degrees in neutral geometry (that is, geometry that doesn't depend upon the parallel postulate). In studies of the foundations of geometry, interesting results have been obtained by denying the Archimedean axiom that states:

If AB and CD are two segments, then there exist on the line AB a number of points

$A_1, A_2, \ldots A_n$ such that the segments $AA_1, A_1A_2, A_2A_3, \ldots, A_{n-1}A_{n\,a}$ are congruent to CD and such that B lies between A and A_n.

In the resulting non-Archimedean geometry there are segments such that the multiple of the first one by any whole number, however large, need not exceed the second segment. Dehn obtained many interesting results in this geometry; among them being the existence of similar but noncongruent triangles and the fact that through a given point an infinite number of lines can be drawn that are parallel of some given line not containing the point.

At the International Congress of Mathematicians in Paris in 1900, David Hilbert set forth 23 problems that he felt mathematicians should solve over the course of the 20th century. The first to be solved was the third problem, which can be stated as follows:

> "In two letters to [Christian] Gerling, Gauss expresses his regret that certain theorems of solid geometry depend upon the method of exhaustion … Gauss mentions in particular the theorem of Euclid that tetrahedra of equal altitudes have volumes that are proportional as their bases. … Gerling succeeded in proving the equality of volume of symmetrical polyhedra by dividing them into congruent parts. Nevertheless, it seems to me probable that a general proof of this kind for the theorem of Euclid just mentioned is impossible, and it should be our task to give a rigorous proof of its impossibility. This would be obtained as soon as we specify two tetrahedra of equal bases and equal altitudes which can not be split up into congruent tetrahedra and which cannot be combined with congruent tetrahedra to form two polyhedra which themselves could be split up into congruent tetrahedra."

The problem is a natural extension of the question regarding two polygons that are "scissors congruent" (also known as "equidecomposable"), that is, when one of the polygons can be cut into polygonal pieces and arranged to form the second. Clearly, if two polygons are scissors congruent, they must have the same area. The converse: "If two polygons have the same area, then they are scissors congruent," is known as the Wallace-Bolyai-Gerwien Theorem named for William Wallace, Wolfgang Bolyai and P. Gerwien, who each proved the result in the early 1800s. The extension of this problem is whether any two polyhedra with the same volume are always "scissors congruent." Dehn negatively answered Hilbert's question only a few months after it was posed. The rather ordinary tetrahedra in Figure 11.8, whose vertices are respectively [(0,0,0)), (1,0,0), (0,1,0), (0,0,1)] and [(0,0,0), (1,0,0), (0,1,0), (0,1,1)] serve as a counterexample. The two tetrahedra have the same volume, but are not scissors congruent.

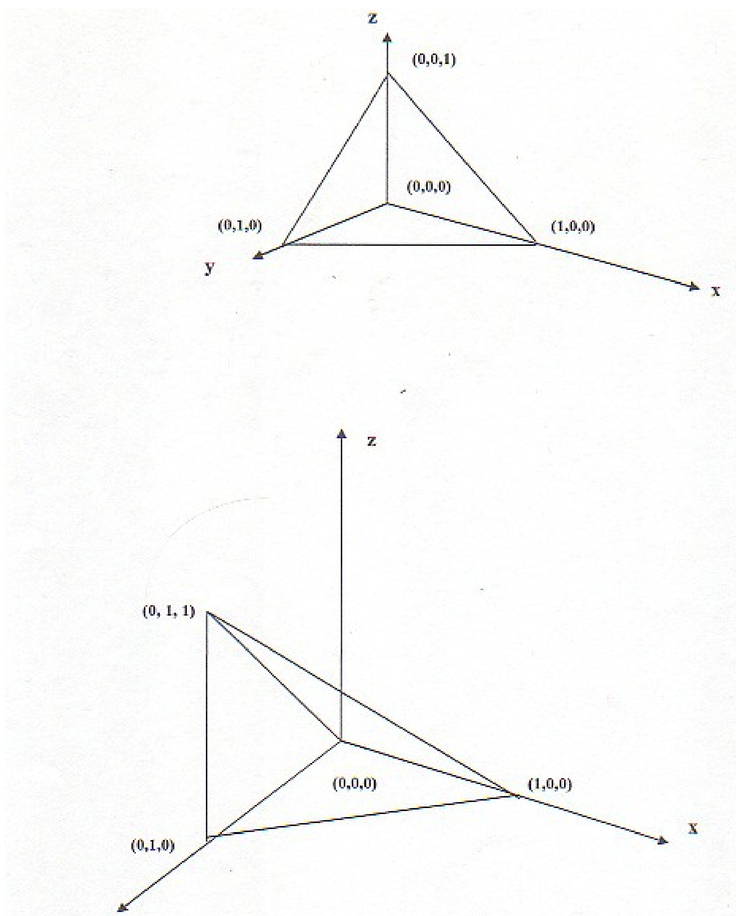


Figure 11.8

Dehn was a *Privatdozent* at Münster from 1901 to 1911. The next two years he was extraordinary professor at Kiel and then from 1913 to 1921, ordinary professor at Breslau. In 1912 he married Toni Landau, with whom he had three children. During WWI he served in the army and afterwards he became professor of pure and applied mathematics at the University of Frankfurt, succeeding Ludwig Bieberbach, but he lost his position due the Nazi anti-Semitism laws in 1935. He remained in Frankfurt for three more years, but wisely sent his children out of Germany. On November 11, 1938, the morning after the infamous *Kristallnacht* ("Night of Broken Glass"), when members of the Nazi party killed 91 Jews, left hundreds seriously injured, gutted some 7500 Jewish business and demolished 177 synagogues, the Dehns and about 30,000 other prosperous Jews were arrested and were offered their release only if they emigrated and surrendered their wealth. Dehn and his wife made it to Denmark and then to Norway, but when the Germans invaded Norway in March 1940, they were again in danger of arrest. With the help of Ernst Hellinger and other former colleagues who had escaped to America, the Dehns traveled to the United States, by a circuitous route through Scandinavia, Russia and Japan, because crossing the Atlantic was too risky due to German U-boats. They joined thousands of other Holocaust refugees who endured the 9200-kilometer trip from Moscow to Vladivostok aboard the trans-Siberian railway. Although Dehn developed a life-threatening combination of flu and pneumonia, he recuperated somewhat when they arrived by ship in Japan, before making the final leg of their trip across the Pacific, arriving at San Francisco on New Year's Day, 1941.

Despite his mathematical eminence, Dehn was unable to find any permanent position. He briefly taught at the University of Idaho (Southern Branch), Illinois Institute of Technology and St. John's College in Annapolis, Maryland. Finally in 1945 he secured a position at an institute not noted for mathematics, Black Mountain College, which had no accredited degrees and a curriculum mainly in the creative arts. There were no trained mathematicians on the staff when Dehn was invited to give two lectures at the College in 1944. Trying to relate to his audience he chose as his topics "Common roots of mathematics and ornamentics" and "Some moments in the development of mathematical ideas." He was offered a permanent appointment at $25 a month, but negotiated his salary upwards to a magnificent sum of $40 a month. Dehn remained at Black Mountain College, until his death of a coronary embolism in 1952, the only mathematician the school ever had. He was buried in the woods near the college at a spot marked by a tablet made in the college's pottery shop. The college closed four years later. Some may consider it a shame that Dehn was compelled to finish his career at a small Southern college where mathematics was an after thought to the arts. Still, he was almost 67 when he took the position and no prestigious mathematical institution made him any offer of a permanent post.

Black Mountain College was the joint creation of a handful of radical American educators and persecuted German professors from the Bauhaus. Mississippi-born Rhodes Scholar and irreverent radical John Andrew Rice lost his position at Rollins College in Florida for his attacks on the administration. During the Depression, joined by a few sympathetic colleagues and students, Rice set about to bring to fruition his dream of an ideal university built around a community of artists. Meanwhile in Germany, the Bauhaus closed rather than submit to Nazi influence, leaving Josef Albers and other professors with no future in their homeland. The combination of Rice, Albers and others resulted in the establishment of the eccentric Black Mountain College. Its motto might well have been the broken English statement of intent Albers made on his arrival: "I come to make open the eyes." Students and teachers dined together and shared chores such as kitchen work, farming, building and garbage collection. Black Mountain played a key role in rescuing many refugee European scholars from the Nazi menace. Besides Dehn, it became home to physicist Peter Bergmann until he moved on to Princeton to become Einstein's assistant.

One of Dehn's colleagues at Black Mountain College was Buckminster Fuller who taught there in the summer sessions of 1948 and 1949, during which time he worked on his geodesic dome. Other artistic and academic pioneers, who found Black Mountain College an inspirational place to break new ground, were painters Josef and Anni Albers, Elaine Field de Kooning, Willem de Kooning, Robert Motherwell, and Franz Kline, sculptors Richard Lippold and Peter Grippe, filmmaker Arthur Penn Warren, interior designer Richard Lischer, art historian Beaumont Newhall, Harvard architecture professor Charles Burchard, poets Edward Dorn, Robert Creeley, Denise Leverton and Robert Duncan, writer and social critic Paul Goodman, composer John Cage and choreographer Merce Cunningham.

Quotation of the Day: “Mathematics is the only instructional material that can be presented in an entirely undogmatic way.” – Max Dehn