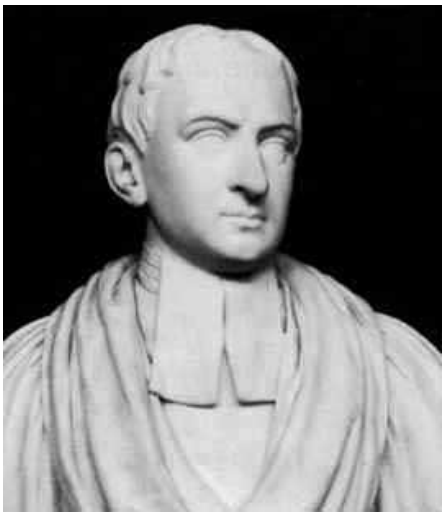
Roger Cotes



Roger Cotes (July 10, 1682 – June 5, 1716) was one of the most ardent of the early Newtonians. He did important work in mathematics and astronomy and edited the second edition of Isaac Newton's *Principia Mathematica*. His work was significant but he did not realize his full potential because he died at age 33, leaving an unfinished series of imposing researches on optics and a large number of other unpublished manuscripts. Newton reputedly said of his young friend, "If Cotes had lived, we might have known something." Cotes was born near Leicester, where he got his earliest education, and showed impressive talent for mathematics. He attended St. Paul's School, London before matriculating at Trinity College, Cambridge, where he was elected a Fellow.

When classical scholar Dr. Richard Bentley became Master of Trinity in 1700, he wanted to make his college a center for astronomy and natural philosophy. Four years later the Reverend Thomas Plume, Vicar of Greenwich, bequeathed £2000 to establish a chair in astronomy and experimental philosophy, and to build an observatory at Cambridge. Bentley arranged for the Trinity gatehouse to be the site of the new observatory. He then had 24-year-old Cotes appointed the first Plumian Professor of Astronomy and Experimental Philosophy. Cotes took holy orders in 1713, a prerequisite for a professorship at Cambridge. Newton, who had numerous conflicts with John Flamsteed, the first Astronomer Royal at the Royal Greenwich Observatory, wanted the new observatory to compete with that of Flamsteed. Even before its founding, Cambridge owned several astronomical instruments for positional measurement. Thereafter, a number of additional astronomical and mathematical instruments were obtained, commissioned from important London instrument makers.

Strangely Newton chose Bentley to publish the second edition of the *Principia*. The latter was no scientist or mathematician, and there were many better choices for the job, scientists who would have jumped at the chance. When asked later why he chose Bentley, Newton replied that the Master of Trinity "was covetous & loved money & therefore I let him that he might get money." Records show that Bentley made a profit of £200 from the sale of the books. To protect his investment Bentley wisely chose Cotes to do the actual editing of the great work and the brilliant young man took the assignment very seriously. Now and then his zeal for his assignment annoyed Newton, because Cotes found fault with some of Newton's arguments and uncovered numerous errors in the copy that took a long time to resolve.

Cotes spent much of his time from 1709 to 1713 preparing this second edition, essentially in the form known today. He wrote an outstanding preface for the work, beginning by explaining the "manner of philosophy" used. In essence Cotes' exposition is an early explanation of the *scientific method*. He summarized the arguments needed to deduce "the Principle of Gravity from the Phenomena of Nature…" The notions of "action of a distance" and gravity as an innate property of matter, commonly attributed to Newton, were first clearly articulated by Cotes.

Cotes published only one paper during his lifetime, "Logometria", which appeared in the *Philosophical Transactions of the Royal Society* (1714). In it he developed the theory of logarithms, which he defined as "measures of ratios", and applied them to the integration (or in the language of the time, "finding the fluents of fluxional expressions") of a variety of rational and irrational algebraic forms. The memoir ended with the rectification of a number of curves and the evaluation of the surface areas and volumes of the associated solids of revolution. William Jones encouraged Cotes to publish more of his work with the *Transactions*, but he wished to publish with the *Cambridge University Press* and died before

he could do so.

After his death most of his writings were collected and published by his cousin Robert Smith, who succeeded Cotes as Plumian Professor. Smith, who founded the Smith Prize at Cambridge, called the collection *Harmonia Mensurarum* (1722) and issued it in four parts. The work is among the early works to recognize the periodicity of the trigonometric functions. In it Cotes clarified the concept of the logarithm of a complex number by proving a theorem on complex numbers, which in modern notation is given by $\ln [\cos \theta + i \sin \theta] = i\theta,$ and at the time did not attract much attention. This equation is usually attributed to Leonhard Euler who rediscovered Cotes' theorem in 1740 and gave it in modern exponential form.

Cotes anticipated the method of least squares and discovered a method of integrating rational fractions with binomial denominators. He worked out in detail Newton's notions of numerical integration and constructed the table of coefficients now known as Cotesian coefficients or Cotes numbers. His work with framing an early theory of errors, in formulating a method for determining the most probable result from a number of observations, was published in the *Opera Miscellanea* (1722). The collection also contained essays on Newton's *Methodus Differentialis*, on the construction of tables by the method of differences, on the descent of a body under gravity, on the cycloidal pendulum, and on projectiles. In 1750, Thomas Simpson published some of the work that Cotes had hoped to put into print with the Cambridge University Press in two volumes as *The Doctrine and Application of Fluxions.*

Quotation of the Day: "Those who assume hypotheses as first principles of their speculations … may indeed form an ingenious romance, but a romance it will be." – Roger Cotes