Bonaventura Cavalieri

Italian priest and mathematician **Bonaventura Francesco Cavalieri** (1598 – November 30, 1647) developed the method of *indivisibles* for finding areas and volumes. Indivisibles are difficult to explain precisely. We can think of them as in some sense the things from which continuous substances are constructed. A point was the "indivisible" of a line, that is, Cavalieri considered a line to be composed of an infinite number of points. A plane is composed of an infinite number of indivisibles, namely lines, and squares of plane circles were the "indivisibles" of a pyramid, cone, etc. In Cavalieri's treatment, a moving point generated a line; a moving line generated a surface; and a moving surface generated a solid. Cavalieri was not the first person to consider geometric figures in terms of the infinitesimal. It had already been incorporated into medieval scholastic philosophy. Cavalieri was not the first to use such a concept in computing areas and volumes, but he was the first one who published a work on the concept. His work provided a deeper notion of sets, namely that it isn't necessary to assign elements to a set; it is enough that there exists some precise criterion for determining if an element does or does not belong to a set.

Born Francesco Cavalieri in Milan, he took the name of his father, Bonaventura, when he entered the Congregation of Hieronymites or Jesuati (not Jesuit) order when he was thirteen. The order was established in 1367 to care for and bury the victims of the Black Death, the plague that killed more than one-fourth of Europe's population. He received minor orders in 1615, and the next year he was transferred to the Jesuati monastery at Pisa. There he studied philosophy and theology and was introduced to geometry, to which he devoted the rest of his life. He quickly absorbed the works of

Euclid, Archimedes, Apollonius and Pappas. Cavalieri became an accomplished mathematician and one of the most illustrious disciples of Galileo. During his lifetime he wrote at least 112 letters to Galileo, although only two letters from Galileo to Cavalieri are known to have survived. In 1620 Cavalieri was recalled to Milan, where he became a deacon to and protégé of Cardinal Borromeo. Cavalieri lectured on theology for three years at the monastery of San Girolamo in Milan and then was made prior of St. Peter in Lodi, and in 1626, prior of the Jesuati monastery in Parma. He vainly hoped to be appointed lecturer in mathematics at the University of Parma. In 1629, through the intervention of Galileo, he secured the chair of mathematics at the University of Bologna, a position he held until his death in 1647.

Cavalieri was the first to recognize and popularize the value of logarithms in his *Directorium generale uranometricum* (*A General Directory of Uranometry*, 1632). Uranometry is the science of the measurement of the positions, magnitudes, distances, etc. of the stars. The tables of logarithms that he published included logarithms of trigonometric functions for use by astronomers. However, his greatest contribution was his principle of indivisibles – a procedure that further developed Archimedes' method of exhaustion, although Cavalieri would not be aware of Archimedes' work. Cavalieri announced the principle in 1629, but it did not appear in print until six years later in his treatise *Geometria indivisibilibus continuorum*. His theory was spurred by Kepler's *Stereometria* and by the encouragement of Galileo. The main advantage of the method of indivisibles was that it was more systematic than the method of exhaustion. In effect, Cavalieri found a result equivalent to evaluating the integral

$$\int_0^a x^n dx \quad \text{as} \quad a^{n+1}/(n+1)$$

Cavalieri's method of indivisibles forms a crude type of integral calculus in which an area is thought of as consisting of lines and that a solid's volume can be regarded as composed of areas that are indivisible. With his theory he was able to solve many problems connected with the quadrature of curves and surfaces, finding of volumes, and locating centers of mass, all of which were superseded at the beginning of the 18th century by the integral calculus. For instance, he determined that a pyramid or cone has 1/3 the volume of the prism or cylinder of equal base and height. He didn't actually find the area of a figure or the volume of a solid as being so many "square units" or "cubic units;" instead he determined the ratio between the required area or volume with that of some other easily calculated area or volume.

Cavalieri did not rigorously develop his theory of indivisibles, but he did not view this as an important defect. He was intent on finding some relatively simple practical method for finding areas and volumes. He was not concerned with the Zeno's puzzling paradoxes that haunted inquiries into infinite processes. He and other mathematicians of the period ignored the logical imperfections in his use of infinitesimal quantities. They developed methods whereby whenever a quantity changed in value according to some continuous law, as most things in nature seemed to do, the rate of increase or decrease in such a change was measurable. Later, when these logical imperfections were removed, mathematicians developed infinitesimal calculus, enabling scientists to pry loose the secrets of nature that for so long had been a closed book.

Quotation of the Day: "Rigor is the concern of philosophy not of geometry." – Bonaventura Cavalieri