BHASKARA

Bhaskara (1114 – c. 1185) called *Acarya* ("the Learned"), who was born in Bijjada Bida, India, near the Sahyadri Mountains, was the most distinguished mathematician of the 12th century. He is credited with the introduction of concepts found much later in the development of the calculus. Bhaskara is said to have been the lineal successor of the 7th century mathematician Brahmagupta as head of the astronomical observatory at Ujjain, the leading mathematical center of ancient India. Brahmagupta studied the instantaneous motion of planets, particularly those of the Moon, developing a formula that was used in predicting future eclipses. Bhaskara extended the results, producing what seems to be the differential of the sine. In his most famous work, *Siddhantasiromani* (Head Jewel of Accuracy), he introduced an "infinitesimal" unit of measure, the *truti*, which is 1/33,750 of a second. Apparently Bhaskara only viewed this as a tool in his astronomical work and there is no evidence that he saw it as the threshold of a new branch of mathematics.

The *Siddhantasiromani* is divided into four parts; *Lilavati* (The Beautiful), an arithmetic text named for his daughter, *Bijaganita* (Seed Counting), a work on algebra, *Goladhyaya* (Celestial globe) and *Grahaganita* (Mathematics of the Planets). In this work Bhaskara gave the first systematic exposition of the decimal system. He also filled in many of the gaps of the work of Brahmagupta, including finding a general solution to what in the 17th century came to be known as Pell's equation. The Arabs knew Bhaskara's work but they failed to utilize or extend many of his discoveries. In 1817, H.T. Colebrooke translated Bhaskara's work into English.

As was the custom of the time, Bhaskara wrote his learned treatises in verse, although he did add

explanations in prose. It also was customary for books to commence with a salutation to the god of wisdom. *Lilavati* begins:

> "Salutation to the elephant-headed Being who infuses joy into the minds of his worshippers, who delivers from every difficulty those who call upon him and hose feet are reverenced by the gods."

Many problems in *Lilavati* were addressed as questions to his beloved daughter, as in the following:

> "Fawn-eyed child Lilavati, tell me, how much is the number [resulting from] 135multiplied by 12, if you understand multiplication by separate parts and by separate digits. And tell [me], beautiful one, how much is that product divided by the same multiplier?"

Bhaskara anticipated the modern rules of signs (minus times minus is a plus, minus times plus is a minus), and was the first to develop some understanding of what it meant to divide by zero. Bhaskara used letters to represent unknown quantities and solved general equations of the first and second degree. His other works includes *Karanakutuhala* (Calculation of Astronomical Wonders), based on his astronomical observations of planetary positions.

In her delightful book *The Mainstream of Mathematics* (1955) Edna Kramer reported the following sad tale. Astrologers predicted that Lilavati would never marry. Bhaskara, no slouch as an astrologer, calculated a certain day at a very certain time that she could be married, but no other time would do if it passed. As the guests gathered for the ceremony on the lucky day, the Learned One and his family eagerly watched a special water clock that Bhaskara had devised which would proclaim the happy instant for the marriage to take place. The excited bride leaned over the instrument but did not notice that a pearl from her headpiece fell into the timepiece plugging up the orifice through which the water flowed. By the time the mishap was discovered the appointed time had passed. The wedding went on as planned, but her husband died soon after the ceremony. To console his daughter, who remained a

widow the rest of her life, Bhaskara promised to name a book after her that "will last to the latest times … for a good name is a second life and the groundwork of eternal existence." Thus was his book *Lilavati* named.

In *Lilavati*, Bhaskara featured a pictorial proof of the Pythagorean theorem. Although the ancient Chinese knew the same diagram proof, it is generally credited to Bhaskara. The diagram consists of a square of side c with an inner square of side $b - a$, where a and b are the sides of the four right triangles whose hypotenuse is c [Figure 3.1]. His one word proof was "Behold!" Bhaskara's proof is also a dissection proof.

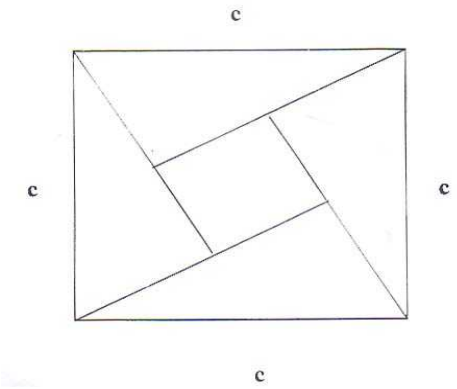

Figure 3.1

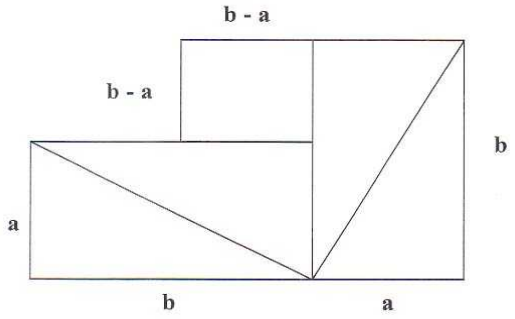

Figure 3.2

The pieces of the large square are fitted together to form Figure 3.2, which consists of the inner square of Figure 3.1 and the four right triangles each with sides a and b and hypotenuse c. The area of the large square, which is c^2 is equal to the sum of the areas of the small square and the four right triangles. That is,

$$c^2 = 4\,(1/2\,ab) + (a - b)^2 \;=\; 2ab + a^2 - 2ab + b^2 = \; a^2 + b^2$$

Quotation of the Day: "A particle of tuition conveys a science to a comprehensive mind: and

having reached it, expands of its own impulse. As oil poured on water, as a secret entrusted to the vile, as alms bestowed upon the worthy, however little, so does science infused into a wise mind spread by intrinsic force." – Bhaskara