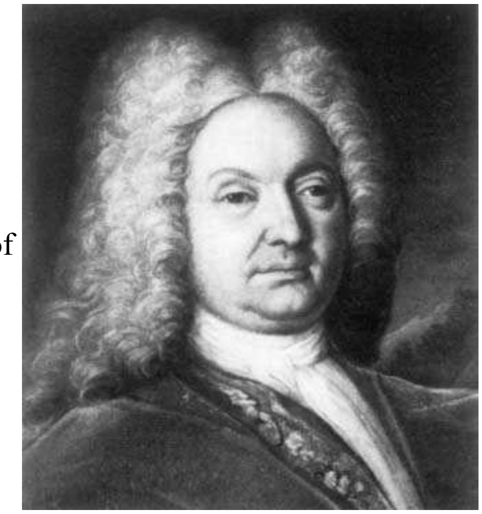
Johann Bernoulli



Another member of the remarkable Bernoulli family is **Johann Bernoulli** (July 27, 1667 – January 1, 1748). He was a pioneer in the field of calculus and applied the new techniques to applied problems. He wrote on differential equations, finding the lengths of curves and the areas enclosed by curves, isochronous curves, and curves of fastest descent. Sometimes referred to as Johannes, Jean and John, depending upon the translation, he was born and died in Basel, Switzerland. His merchant father intended for him to enter the spice business but like his older brother Jacob, he had no interest in commerce. By the time his father reluctantly allowed him to enter the University of Basel to study medicine (despite his preference for mathematics), Jacob, who was twelve years older than Johann, held the university's chair of mathematics. Johann studied mathematics with Jacob and after two years was his elder brother's equal in mathematical skill, a fact that did not please the older man. Jacob continued to refer to his brother as his "pupil," a belittlement that Johann detested. The two had bitter arguments and engaged in public criticism of each other's work.

In 1691, Johann moved to Geneva where he lectured on the differential calculus, being one of only a handful of individuals in the world (including his brother Jacob) to have sufficient knowledge of the new mathematics to do so. In 1694, he wrote a paper on the movement of muscles, which became the first application of the principles of differential calculus to a biological problem. From Geneva, he went to Paris where he met the Marquis de l'Hôpital, at the time perhaps the best mathematician in France, although no match for Bernoulli. The nobleman paid Bernoulli a handsome fee to teach him the new calculus. Johann was enraged when l'Hôpital published the first calculus text *Analyse des infiniment petits pour l'intelligence des lignes courbes* (1696), based on Bernoulli's lessons, a fact not acknowledged by l'Hôpital. It is almost certain that the result which has come to be known as" l'Hôpital's rule" was the work of Bernoulli. After the Frenchman's death in 1704, Bernoulli publicly protested that he was the author of the calculus book, but was not believed. In 1922, a copy of Bernoulli's course, made by his nephew Nicolaus (I), was found in Basel. It was virtually identical with l'Hôpital's book, but in fairness it must be acknowledged that l'Hôpital corrected a number of Bernoulli's errors.

In 1695 Bernoulli was appointed to the chair of mathematics at Groningen, where he unhappily spent ten years. He was involved in a number of religious disputes, including the accusation that he was a follower of Descartes' philosophy, unacceptable to the Calvinists. As he was returning to Basel to accept the Greek chair, Johann learned of his brother Jacob's death. He became Jacob's successor as professor of mathematics, serving in the post from 1705 until his death in 1748. He was among the most successful teachers of his age, inspiring his pupils with a passionate zeal for mathematics that he felt himself. Among his students and disciples including three of his sons, were P.L.M. Maupertuis, Gabriel Cramer, Alexis Claude Clairaut, and most notably Leonhard Euler. Johann expelled his son Daniel from his house for obtaining a prize from the French Academy that he had expected to receive.

In 1713 Johann became involved in the Newton-Leibniz controversy. A strong supporter of Leibniz, his criticism of Brook Taylor's *Methodus incrementorum* was an attack upon Newton's method of fluxions. It was due to Bernoulli's influence that the differential notation of Leibniz rather than the fluxional notation of Newton was adopted in continental Europe. Among Johann's chief discoveries were the exponential calculus, the treatment of trigonometry as a branch of analysis, the conditions for a geodesic, the determination of orthogonal trajectories, and the solution of the brachistochrone problem. He coined the word *brachistochrone* (from two Greek words *brachistos* for "the shortest" and *chronos* for "time") for the path that a particle will slide down without friction from one point to another in the shortest time. Johann announced in the *Acta Eruditorum* in June of 1696 that he had found an excellent solution of the problem but wouldn't publish it at the time in order to allow other mathematicians to solve it and similar problems. Newton, Leibniz, de l'Hôpital, and Jacob Bernoulli solved the brachistochrone problem by determining that the required curve is an arc of a cycloid. Johann's resolution of the problem he proposed was to solve the analogous problem of finding the path of light refracted by transparent layers of varying density. Problems similar to the brachistochrone led to the development of a branch of calculus known as the "calculus of variations."

When Johann issued the *brachistochrone* challenge to the mathematical world, Newton solved it the same evening he received it and sent his solution anonymously to Bernoulli. The latter knew it came from Newton, because as he said, "I recognize the lion by his paw." It was common practice at the time for mathematicians to challenge others to solve problems, often ones they had already solved. The same year as Johann's challenge, Jacob challenged the world, but particularly his younger brother, to solve a problem in which a curve had to be determined that would give a maximum or minimum area when each of its y-coordinates was a given function of the corresponding y-coordinate of another curve. When Jacob declared Johann's solution was incorrect, the latter, who had a violent temper, exploded, prompting a bitter quarrel between the two brothers. After Jacob's death, Johann produced a solution to the problem that he claimed to be his own, but in fact was really his brother's. Outraged at what he felt to be a theft of his work and ideas by l'Hôpital didn't prevent him from doing likewise.

Johann also made important contributions to mechanics and the theory of kinetic energy. He seems to have been the first one to use the word "function" in a 1698 publication about curves. He certainly was the first to define a function as an analytic expression. He proposed the Greek letter φ (phi) be used as notation for a function of x, that is $\varphi(x)$. His pupil Euler introduced f for a function and expressed it as f(x). Known as the "Archimedes of his Age," Johann was elected a fellow of the academies of Paris, Berlin, London, St. Petersburg and Bologna. He was a great correspondent, exchanging 2,500 letters with 110 scholars. With the deaths of Leibniz and Newton, Johann was acknowledged as the foremost mathematician of his time.

Galileo was the first to show that the path of a projectile, disregarding wind resistance, is a parabola. He mistakenly thought he had found another application of the parabola in the curve assumed by a rope or a chain (catena), fixed at two points, hanging under its own weight. But this curve, first called a catenary in a letter from Christiaan Huygens to Leibniz, is not only not a parabola; it isn't even represented by an algebraic function. Joachim Jungius detected Galileo's mistake but the true form of the curve was not discovered until Jacob posed the problem of the catenary (Figure 7.5) in the *Acta Eruditorum*. Leibniz, Huygens and Johann all solved the problem, Huygens by geometric means, and Leibniz and Bernoulli by the new means of the calculus. Its equation in rectangular coordinates is $\cosh(x) = (e^x + e^{-x})/2$.

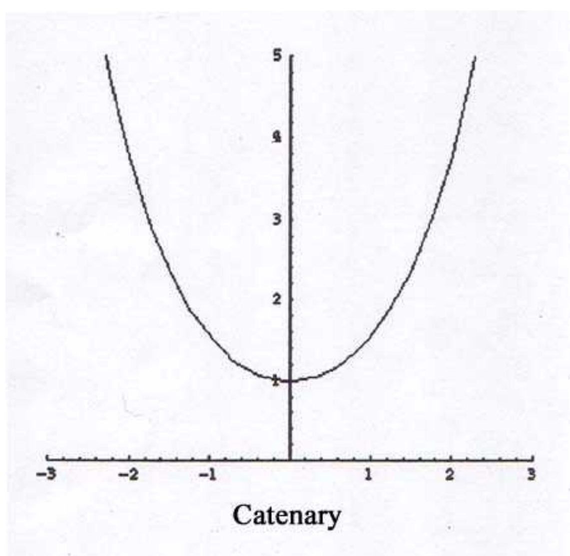


Figure 7.5

Johann took great delight in the fact that he had been able to solve the problem and his brother Jacob had not. In a letter to Pierre Rémond de Montmort of September 29, 1718, he gleefully recalls his triumph of 27 years earlier:

> “The efforts of my brother were without success; for my part, I was more fortunate, for I found the skill (I say it without boasting, why should I conceal the truth?) to solve it in full and reduce it to the rectification of the parabola. It is true that it cost me study that robbed me of rest for an entire night. It was much for those days and for the slight age and practice I then had, but the next morning, filled with joy, I ran to my brother, who was still struggling miserably with this Gordian knot without getting anywhere, always thinking like Galileo that the catenary was a parabola. ‘Stop! Stop!’ I said to him, ‘don’t torture yourself any more to try to prove the identity of the catenary with the parabola, since it is entirely false. The parabola indeed serves in the construction of the catenary, but the two curves are so different that one is algebraic, the other is transcendental.’”

Children’s jump ropes form catenary curves. The Gateway Arch in St. Louis is based on an inverted catenary - 192 meters high and 192 meters wide at its base. In suspension bridges, neither the cable nor the deck is greatly heavier than the other, so the actual curve used is something between a catenary and a parabola. The catenary is not a family of curves; it has only one shape. If two ends of a chain are held, and the distances of its endings are varied, one merely sees different scales of the catenary. Every segment of a hanging chain pulls on every other segment, giving it its shape. Near the points of suspension, the chain is almost vertical because this part of the chain has the most weight pulling downward. Near the bottom of the chain, its slope decreases because the chain is supporting less and less weight. Stan Wagon, a Macalester College mathematics professor built a bicycle with square wheels that when driven on a road made up of pieces of an inverted catenary rides as smoothly as a normal bike on a flat surface.

Quotation of the Day: "But just as it is easy to find the differential of a given quantity, so it is difficult to find the integral of a given differential. Moreover, sometimes we cannot say with certainty whether the integral of a given quantity can be found or not." – Johann Bernoulli