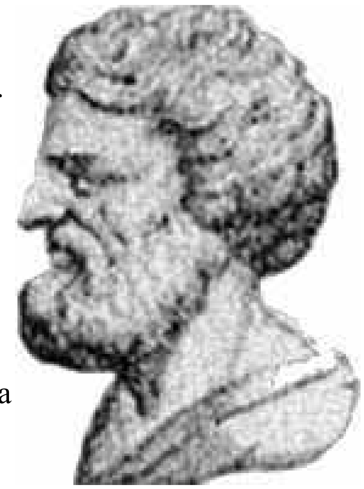
Apollonius



Apollonius of Perga's (c. 262 BCE – c. 190 BCE) mathematical powers were so extraordinary that he became known as "the great geometer." Other than the *Conica* ("On Conics"), one of the greatest scientific works of the ancient world, most of his work has been lost. Though the majority of his investigations were in geometry, he also explored optics with his study *On the Burning Mirror*, in which he showed that parallel beams of light hitting a spherical mirror do not converge to a point, which was commonly believed in his day. Most of his contributions to astronomy are unknown, although he is credited with the hypothesis of eccentric orbits, that is, a system of circles called deferents and epicycles used to suggest a way of explaining the apparent motion of the planets and the varying speed of the moon.

Apollonius was born in Perga in the Ionian kingdom of Pamphilia in southeastern Asia Minor in what is now Turkey and died in Alexandria, Egypt. It is because of his work, and that of Euclid and Archimedes, that the period from around 300 to 200 BCE is referred to as "the Golden Age of Greek mathematics." There were many Greek scholars named Apollonius, so it is necessary to identify him as "Apollonius of Perga." Little is known of his life except that he was born about 25 years after Archimedes and about 50 years after Euclid. Most information about him has been inferred from his personal letters that form the prefaces to the various books of his monumental treatise *Conica*. According to Pappus, as a young man Apollonius studied with the successors of Euclid at Alexandria and probably lectured there. Later he taught at Ephesus and Pergamum, where a university had been established meant to rival the Academy at Alexandria. It was at Pergamum that he met Eudemus and Attalus, each of whom received copies of his books on conics, accompanied by an explanatory letter.

The *Conica* consisted of eight books containing 387 propositions. It was such a complete study of the conic curves that it left little for anyone to add to the subject. Some historians conjecture that many of the properties Apollonius discussed were derived from a type of coordinate geometry where the diameter of a conic played the role of the *x*-axis and the perpendicular at the vertex served as the *y*-axis. The first four books, forming an elementary introduction to the basic properties of conics, have survived in Greek. Euclid knew most of these results, but Apollonius organized, generalized, and greatly extended the work of the previous century and a half. In the first book, Apollonius demonstrated that the three conics to which he gave the names ellipse, parabola and hyperbola are produced from the same cone [Figure 8.15]. Prior to the *Conica,* the three basic curves had been defined by means of three different types of right circular cones, according to whether the vertex angle of the cone was less than, equal to or greater than a right angle. Another innovation of Apollonius was to replace the single-napped cone with a double-napped cone. In this way he was able to consider the two branches of a hyperbola as two parts of a single curve rather than two separate curves. He also demonstrated that to define the conics, a right circular cone wasn't necessary, an oblique (slanting) circular cone would do just as well.

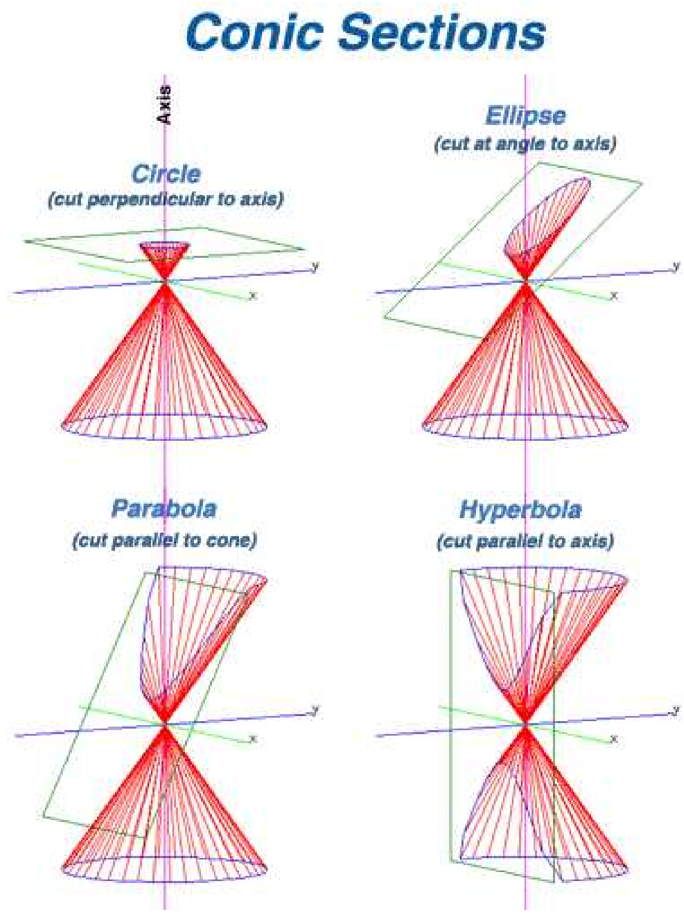


Figure 8.15

In Book two, Apollonius investigated the relationship of hyperbolas with their asymptotes and showed how to draw tangents to the given curves. Book three contained more of the propositions concerning conics that are found in modern textbooks. In Book four, he developed the theory of lines cut harmonically and investigated the points of intersection of systems of conics. The next three books have come down to us in Arabic and the eighth book is now lost. In Book five, Apollonius discussed the number of normals that can be drawn from a point to a conic. He viewed normals to a conic section as minimum and maximum straight lines drawn from a given point to the curve, and tangents as lines perpendicular to normals. It is in this book that he found the center of curvature at any point of a conic, and the evolute of the curve. In Book six, he treated similar conics. The seventh and eighth books discussed conjugate diameters of conics.

Besides the *Conica*, Apollonius wrote numerous shorter treatises but most of these are lost while traces of others have survived in partial Arabic translations. We know something of the nature of these lost works because ancient writers described them. Apollonius' two books *On Proportional Section* survive from an Arabic version that was translated into Latin in 1706. Other works include *Tangencies*, *On Plane Loci*, *On the Cylindrical Helix*, and *Comparison of the Dodecahedron and the Icosahedron*. One work whose title does not clearly reveal its content is *Quick Delivery*. It contained methods for making arithmetical calculations. Apollonius wrote *On the Stations and Regressions of the Planets*, which Ptolemy used in writing the *Almagest*.

In the *Elements*, Euclid showed how to construct a circle that passes through three given non-collinear points, and how to construct a circle tangent to three given straight lines. These are the simplest cases of what is today familiarly known as "The Problem of Apollonius." The problem is given three things, each of which may be a point, a line or a circle; construct a circle that is tangent to each of the three given things. Tangency to a point means that the circle passes through the point. An example of the problem when the three objects are circles is illustrated in Figure 8.16. In accordance with the preference for rigorous Euclidean methods of construction, Apollonius used only a straight edge and compass.

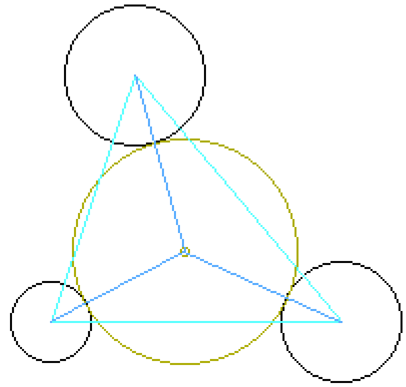


Figure 8.16

Quotation of the Day: “They are worthy of acceptance for the sake of the demonstrations themselves, in the same way as we accept many other things in mathematics for this and no other reason.” – Apollonius’ reply when asked the practical value of his study of the conic sections.