Niels Abel

In his brief life Norwegian mathematician **Niels Henrik Abel** (August 5, 1802 – April 6, 1829) proved that the general equation of the fifth degree (”the quintic’) is not solvable by algebraic means. He was instrumental in establishing mathematical analysis on a rigorous basis, and he revolutionized the theory of elliptic integrals by studying their inverses, the elliptic functions. This fusion of algebra and geometry was based on the formulas used to calculate the circumference of an ellipse. In *Men of Mathematics*, Eric Temple Bell saw Abel’s life as a tragedy and a crime, and not merely because the talented Norwegian mathematician’s life was so short. In Bell’s view it was the indifference and shortsightedness of established mathematicians that caused Abel’s death as much as did his poverty and tuberculosis. It is true that certain leading mathematicians did not make serious efforts to determine the worth of Abel’s work. Still, he did have loyal and devoted friends, among them Bernt Holmboë and August Crelle, who not only recognized Abel’s potential as one of the greatest mathematicians of all time, but did everything in their power to bring the destitute young man to the attention of the mathematical world so he might find his rightful place at the center of it.

Abel was the second of seven children of a poor Lutheran minister in Finnøy, a small island near Stavanger, Norway. Reportedly, he was born three months premature and only survived because he was bathed in red wine. The economic situation in that time in Norway was very arduous, partly caused when its Danish rulers, who had supported France during the Napoleonic wars, were forced to cede Norway to Sweden in 1814. Abel’s father taught him at home until he was 13, at which time he was sent to the Cathedral School in Christiania (now Oslo), which was staffed with inexperienced and

incompetent teachers. Fortunately, Holmboë, an enlightened teacher only seven years older than Abel, replaced the brutish mathematics instructor, who flogged a student so severely that he died. Holmboë quickly recognized his pupil's superior mathematical talents. At age 16 Abel gave the first proof of the general binomial theorem, stated earlier by Newton and Euler, who gave only proofs of special cases.

With Holmboë's encouragement Abel read the works of Euler, Newton, Lagrange, d'Alembert, and Laplace. Later when asked how he had managed to make such progress in mathematics despite his youth, Abel responded, "By studying the masters, not their pupils." One thing that annoyed Abel about the work of the masters was the sloppiness of some of their proofs. Abel spent a good portion of his remaining life patching up the holes in their less than thorough reasoning. His father died in 1820 leaving his family in desperate straits. There was no money to send Abel to a university. He earned a pittance tutoring and doing odd jobs. Holmboë did what he could, even dipping into his own meager resources to allow Abel to continue his studies, and in 1821 he wrangled a scholarship for his pupil at the University of Christiania.

Since the 16th century, mathematicians had tried unsuccessfully to determine the solvability of the general equation of the fifth degree, "the quintic", $ax^5 + bx^4 + cx^3 + dx^2 + ex + f = 0$, by algebraic means. Before entering the university Abel took up the challenge and convinced himself that he had found a solution. He submitted his results to the Danish mathematician Ferdinand Degen for publication by the Royal Society of Copenhagen. While attempting to provide an example of his method, Abel discovered that he was mistaken. This led him to doubt the possibility of an algebraic solution. Changing his tack he proved the impossibility of solving any polynomial equation of degree greater than four. To get his paper published, Abel had to pay for the printing himself. Written in French, the pamphlet consisted of but six pages.

After satisfying the requirements for graduation in a single year, Abel relied on small grants from the University, gifts from friends, and tutoring to provide for his mother and six siblings, while studying on his own. In 1823 he published his first important paper on definite integrals, which included the first ever solution of an integral equation. He also completed another important work on the integration of functions. Had Abel been a citizen of France or Germany these two results would have established his reputation and perhaps led to a university position, but as they were in Norwegian no one read them. Abel sought a royal grant to travel to the mathematical centers of Europe but was forced to wait two years while he struggled to learn a bit of German and French. In the meantime Abel became engaged to Christine (Crelly) Kemp. The same year Abel sent a copy of his work on the insolvability of the quintic to Gauss, who rejected it without reading it as "another of those monstrosities" that aspiring mathematicians often sent him.

Abel traveled to Copenhagen to visit Degen but found he had died. He next went to Berlin where he met Crelle, just then starting the *Journal for Pure and Applied Mathematics*. Crelle instantly recognized that Abel was someone very special in the mathematical world. The first three volumes of Crelle's Journal - as it later became known - contained twenty-two of Abel's memoirs. Abel arrived in Paris in July 1826, but his stay in the City of Lights was a disappointment. The leading figures of the French mathematical world, including Cauchy, showed little interest in an unknown Norwegian with no reputation. Nevertheless, while in Paris, Abel wrote what has come to be considered his masterpiece, his work on elliptic integrals that included what is now known as *Abel's Theorem*. An elliptic integral is one whose integrand consists of a rational function that contains square roots or quartic polynomials. Elliptic integrals originally arose in connection with the problem of finding the arc length of an ellipse and were first studied by Giulio Fagnano and Leonhard Euler. Generally, elliptic integrals cannot be expressed in terms of elementary functions.

Abel gave the first solution of an integral equation and his paper radically transformed the theory of elliptic integrals to the theory of elliptic functions by studying their inverse functions. Abel submitted this work to the French Academy of Sciences. Legendre and Cauchy were chosen by the Academy as referees to judge the paper's worth. A couple of years passed and Abel heard nothing. He shared the contents of the article with Carl Jacobi. The latter wrote to Legendre in an effort to find out what had happened to the memoir. Legendre claimed he could not read Abel's handwriting and suggested that the author submit a new copy. Abel did so, but Cauchy mislaid the second version and forgot about it. The copy was rediscovered in 1830 and was finally printed by the Academy 12 years after Abel's death. Had it not been for inquiries by Jacobi, it might still not have been published. Always generous in his praise of his contemporaries, Jacobi said of Abel's work: "It is above my praise as it is above my own work." In 1830 the French Academy awarded Abel, jointly with Jacobi, the Grand Prize in Mathematics for their work with elliptic functions and integrals. Once again Abel's handwritten two-page paper was misplaced and did not resurface until it was found in Florence in 1952.

While in Paris, Abel lived in miserable conditions, often unable to afford more than one small meal a day. He consulted a physician about his poor health and was diagnosed with tuberculosis. He refused to believe it and left Paris for Berlin, where Crelle was striving to find Abel a position at the University. Abel returned to Norway, hopeful that he would be appointed to the now vacant mathematics chair at the University of Christiania. Instead the governing board threatened to invite a foreigner to take the position if Holmboë did not. The University paid Abel the balance of the grant they owed him and sent students to him to tutor. But he still had his family to support, which left him weakened and sadly aware that he would not live much longer.

Abel traveled by sleigh to be with his fiancée at Froland where she worked as a governess in the home of an English family, the Smiths. During the journey he became critically ill, but recovered sufficiently

to spend a pleasant Christmas with Crelly. His hemorrhaging soon returned and he often slipped into delirium, once shouting “I will fight for my life!” Crelle was alerted to the seriousness of Abel’s illness and redoubled his efforts to secure Abel a position at the University of Berlin. Crelly nursed Abel in his final days and on the morning of April 6, 1829 he died, aged twenty-six years, eight months. Two days later, Crelle wrote that he had finally secured an appointment as professor of mathematics for his friend at the University of Berlin.

While in his final illness, Abel expressed the desire that his friend Baltazar Mathias Kielhau and Crelly should marry. Although the two had never met, Kielhau wrote to Crelly and proposed. She accepted and they became engaged. The couple visited Abel’s grave to pay their respects before they wed in the fall of 1830. Kielhau collected funds that were used to erect a monument on the site of Abel’s grave in Froland churchyard in a plot intended for the Smiths.

Quotation of the Day: “If you disregard the very simplest cases, there is in all of mathematics not a single infinite series whose sum has been rigorously determined. In other words, the most important parts of mathematics stand without a foundation.” – Niels Abel